DERIVING RAINFALL INTENSITY-DURATION-FREQUENCY RELATIONSHIPS FOR KERBALA CITY

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Abstract
Intensity-Duration-Frequency (IDF) relationship of rainfall amount is one of the most commonly used tools in water resources engineering for planning designing, and operation of water resources projects. The objective of this research is therefore, to develop operational IDF relationships for the provinceof Kerbala based on historical data. The IDF curves have been developed considering application to Kerbala area and presented in the form of general mathematical equation, curves relating Intensity–Duration-Frequency of rainfall. The IDF curves are plotted for 2, 5, 10, 25, 50 and 100-year return periods in Kerbala. The values of the constants α, γ and exponents β, τ in the IDF function are calculated from the observed precipitation data by using different methods.Using themethods ofgoodness of fit has been reached in Kerbala that the Log Pearson type III is the best method of other methods.

Keywords:plotting position**,**rainfall frequency analysis.

اشتقاق علاقات الشدة المطرية –الاستدامة–التردد لمدينة كربلاء

الخلاصة
تعتبر علاقة كمية المطر الشدة-الاستدامة-التردد الأكثر استخداما في هندسة الموارد المائية نتيجة لاستعمالها في تخطيط وتصميم وتشغيل مشاريع مصادر المياه. والهدف من هذا البحث الحرص على تطوير العلاقات التشغيلية ل(IDF) لمحافظة كربلاء استنادا على البيانات التاريخية. تم عرض المنحنيات(IDF) المستخدمة في مدينة كربلاء على شكل معادلة رياضية عامة وبشكل منحنيات متعلقة بالشدة - الاستدامة - التردد المطرية. رسمت منحنيات ال(IDF) في كربلاء لفترات عودة ل 2, 5, 10, 25, 50, 100–سنة. حسبت قيم الثوابت في دالة ال(IDF) من رصد البيانات المطرية باستعمال طرق مختلفة. وقد بينت النتائج ان طريقة بيرسون اللوغارتمي النوع الثالث هي أفضل من باقي الطرق باستخدام طرق حسن التوافق.

Introduction
Estimates of high intensity rainfall are very important in rainfall-runoff modeling with respect to water resources engineering, either for planning, designing and operating of water resources projects, or forthe protection of various engineering projects against floods. The rainfall Intensity-Duration-Frequency (IDF) relationship is one of the most commonly used tools in determining design rainfall intensity. Rainfall depths can be further processed and converted into rainfall intensities (intensity = depth/duration), which are then presented in

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IDF curves. Such curves are particularly useful in storm water drainage design because many computational procedures require rainfall input in form of average rainfall intensity.
The establishment of such relationships was done as early as 1932 (see Chow (1988),Dupont and Allen (2006),AlHassoun (2011),and Ibrahim (2012)). Since then, many sets of IDF relationships have been constructed for several parts of the globe. However, such relationships have not been accurately constructed in many developing countries (Koutsoyiannis et al., (1998),and Okonkwoet al., (2010).

Rainfall analyses in the Kerbala basin

Karbala Catchment shown in Figure 1, located between latitudes 32°36'31"N and longitudes 44°01'32"E in central region of Iraq on the eastern edge of the plateau's western desert Euphrates River on the edge of the desert in the middle of the region sedimentary from Iraq. The Karbala city is located 108 km away to the south-west of the Iraqi capital Baghdad, on the edge of the desert in the west of the Euphrates. The conservative climate in general hotin summer, cold in winter and tends to moderation in the eastern part of the terms of the temperature and distribution of rain and humidity, especially in the section located within the area of the plateau. The catchment covers an area of approximately 5034 km^2 and is situated on a plateau of 48 meters above sea level. The area receives one cycle of rainfall that extends from October of the previous year and ends in May of the following year; wherein the dry season runs from June to September.

Data collection and preparation

The data for rainfall (Iraqi weather Bureau, 2013) is analyzed. Figure (2) shows the bar chart of maximum rainfall depth for each year at Kerbala city for the period (1960-2013). There is also great variation in the yearly amount of rainfall. For example, only (12.7) mm of rainfall was measured in 1974 and this rainfall was greater by nearly 17.6 times in 1975. However, the average yearly rainfall during the above period is around (90.52) mm and there is great fluctuation in the amount of rainfall from year to year.

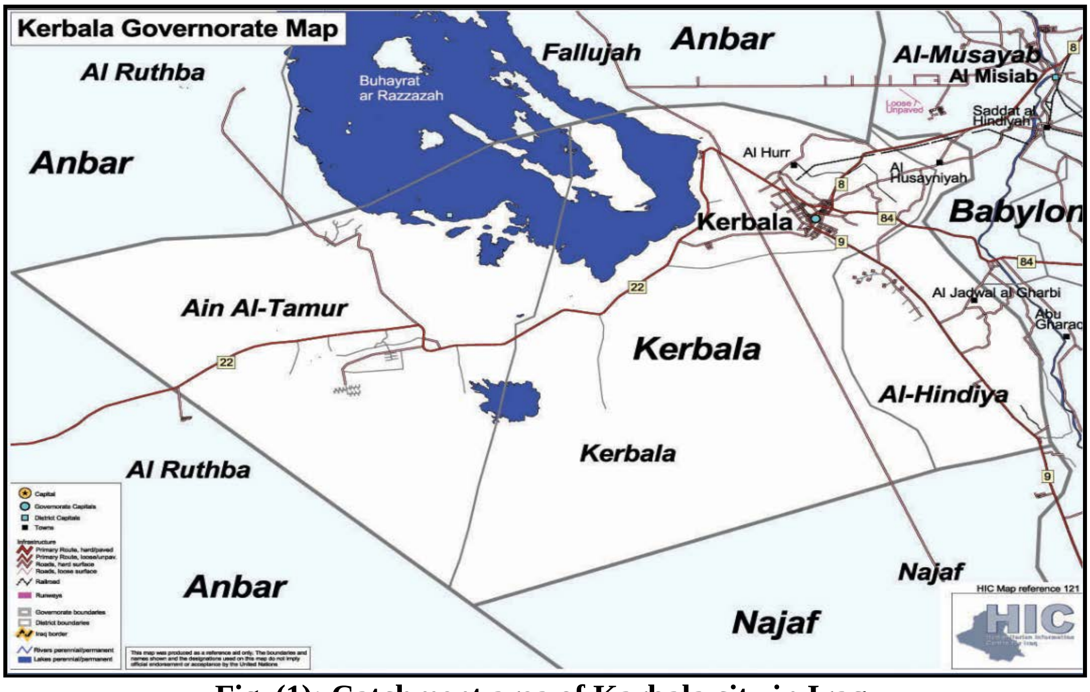

Fig. (1): Catchment area of Karbala city in Iraq.

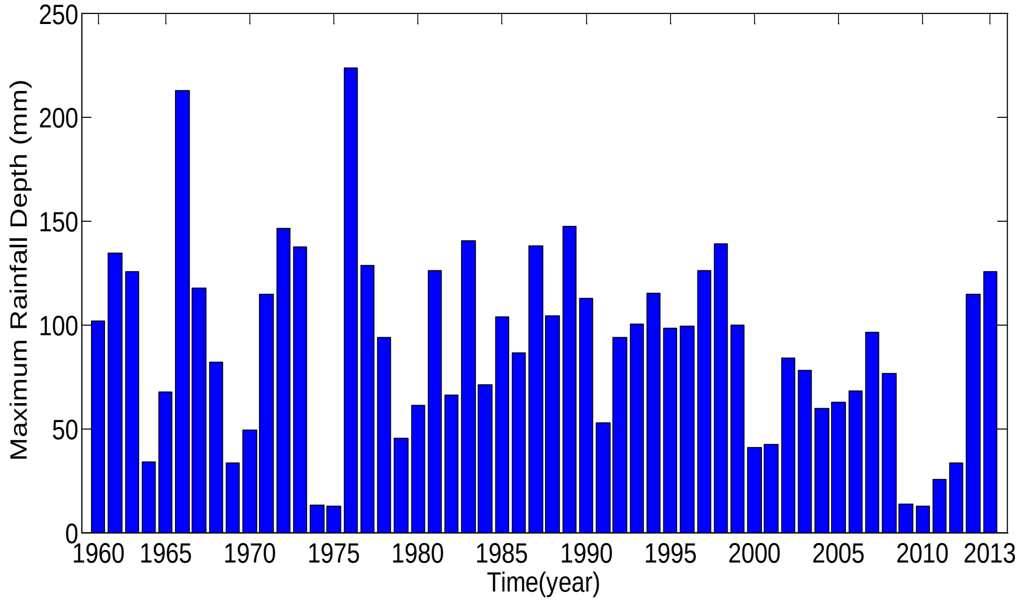

Fig. (2): Maximum Annual Rainfall Depths at Kerbalacity

The mathematical expression of an IDF relationship

The IDF relationship is a mathematical relationship between the rainfall intensity i, the duration D and the return period T (or, equivalently, the annual frequency of exceedance, typically referred to as 'frequency' only). The IDF relation is expressed mathematically as follows (Mohymontet al., 2004):

$$i = f(T, D) \quad (1)$$

$$A = \propto T^{\beta} \quad (2a)$$

$$i = \frac{A}{(D+\gamma)^{\tau}} \quad (2b)$$

$$i = \frac{\propto T^{\beta}}{(D+\gamma)^{\tau}} \quad (2c)$$

I havesuggested an empirical formula to estimate the rainfall intensity in Kerbala region.

$$i = \frac{\propto T^{\beta}}{(D+\omega\gamma^{\varepsilon})^{\tau}} \quad (3)$$

where:
i= The rainfall intensity (mm/hr).
T=The return period (years).
D=The rainfall duration (hr).
$\propto$, ω,γ= Regional coefficients.
β , ε ,τ=Regional exponents.

Frequency of intense storms

The adopted method for determining number of storms of stated rainfall intensity or more within a certain duration and return period according to (Raiford et. al.,2007)are listed below:

a) Sort of all the recorded data for a certain duration with decreasing order (Haan,2002). For example, the recorded data for intensity and 24 hr duration events are given in Table (1).

a) Number of storms of stated rainfall intensity or more within a certain duration is equal to the rank of intensity. For example the rank of intensity equal or greater than 0.42 mm/hr is equal to 14 = number of storms of 0.42 mm/h rainfall intensity or more within a 24 hr duration and the rank of intensity equal or greater than 5.78 is equal to 2= number of storms of 5.78 mm/hr rainfall intensity or more within a 24 hr min duration, see Table (1), and so on. These data are given in Table (2) .

b) State the frequency. If, for example, that the 10-year storm is equalled or exceeded intensity 54/10=4.5 times in 54 years, the generalized time-intensity values may be interpolated from Table (2) by finding for each specified duration the intensity equalled or exceeded by 4.5 storms thatcanbeeasilycalculated as given in Table (3) .Similar calculations for the 2-year, 5-year, 25-year, 50-year and 100-year storms underlie the remaining members of the family of curves in Table (4), and in Figure (3).

Table (1): 120 min Duration Events forKerbalacity.

Rank	Intensity (mm/h)	Rank	Intensity (mm/h)
1	5.85	8	2.93
2	5.78	9	2.42
3	5.55	10	2.26
4	5.25	11	1.97
5	4.84	12	1.30
6	3.97	13	0.84
7	3.76	14	0.42

Table (2): 54 Years Rainfall Record for Kerbalaand Number of Storms of Stated Intensity or More

Dhr	**Intensity (mm/hr)**												
	0.5	**1.0**	**1.5**	**2.0**	**2.5**	**5.0**	**4.5**	**4.0**	**3.5**	**3.0**	**6.0**	**5.75**	**5.5**
1/12	139	132	115	101	96	78	64	53	36	19	15	11	10
1/6	127	118	91	89	82	76	54	48	31	15	12	10	9
1/2	115	99	87	80	76	70	51	34	25	14	11	9	8
3/4	98	90	81	75	67	61	48	24	18	12	10	7	6
1	91	83	67	39	32	30	28	21	14	9	8	6	5
2	82	55	43	33	26	21	19	13	9	7	6	5	4
6	53	39	32	28	13	12	10	9	7	6	5	4	3
12	34	21	16	13	11	9	8	7	6	5	5	3	2
24	14	13	12	11	9	8	7	6	5	4	3	2	1

Table (3): Duration and intensity of the 10-year storm for Kerbalacity.

Duration hr	Intensity mm/hr
1/12	3.12
1/6	2.88
1/2	2.18
3/4	1.87
1	1.73
2	1.29
6	0.84
12	0.55
24	0.31

Table (4): Intensity (mm/hr) for different duration and frequency for Kerbalacity.

Duration	Frequency				
hr	**2**	**5**	**25**	**50**	**100**
1/12	1.41	2.07	3.90	4.57	5.35
1/6	1.39	1.91	3.39	4.16	4.76
1/2	1.25	1.55	2.48	3.28	4.09
3/4	1.16	1.39	2.20	3.05	3.71
1	1.08	1.28	2.01	2.75	3.56
2	0.85	1.00	1.63	2.20	3.12
6	0.46	0.60	1.04	1.40	2.24
12	0.27	0.38	0.66	0.88	1.56
24	0.15	0.19	0.41	0.55	0.83

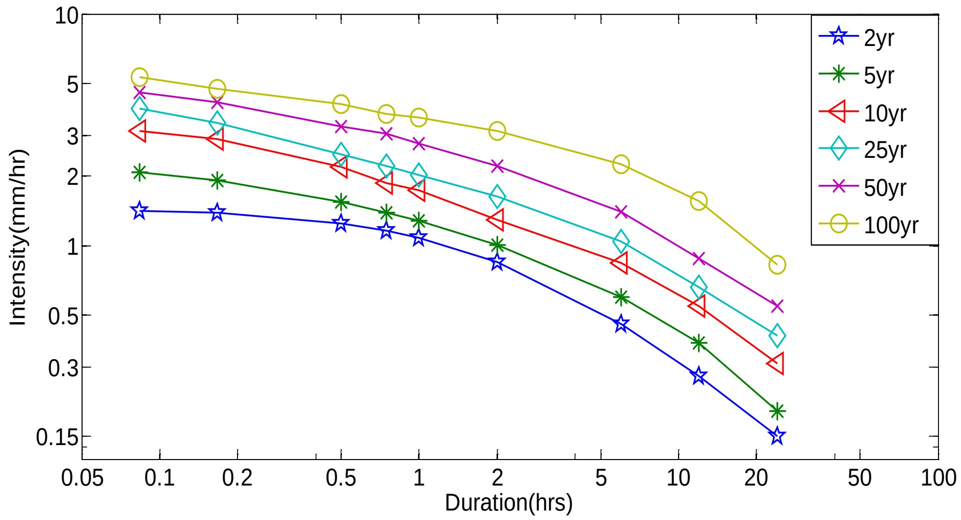

Fig. (3): Intensity-duration-frequency of intense rainfalls for Kerbala.

Methods of parameters estimation

1. Graphical fitting

This method (graphical fitting) does not exist in clear explanation in the references that explain it, therefore it will be explained here. Equation (Eq.2B) can be transformed to read [log i] = log A - τ [log (D + γ)] where the brackets identify the functional scales y= [log i] and x = [log (D + γ)] for direct plotting of i against D on double logarithmic paper for individual frequencies. Straight lines are obtained when suitable trial values of γ are added to the observed values of t. To meet the requirements of equation (Eq.2C) in full, the values of γ and of τ, the slope of the straight line of best fit, must be the same or averaged to become the same at all frequencies. Values of A can then be read as ordinates at (D + γ) = 1, if this point lies within the data, it will be taken. Otherwise it can be brought from the plot. To determine $\propto$ and β, the derived values of A are plotted on double logarithmic paper against T for the frequencies studied. Because [log A] = log $\propto$ + β [logT], the slope of the resulting straight line of best fit equals β, and the value of $\propto$ is read as the ordinate at T =1.

By plotting the values for the 10-year storm taken from Table (3) on double logarithmic paper as explained in Figure (4). Because the high-intensity, short-duration values are seen to bend away from a straight line, they can be brought into line by adding 0.5556 hr to their duration periods, that is, (D+ γ) = (D + 0.5556). Derivation of the equation i =A/(D+ γ) = 2.0664/(D + $0.5556)^{-0.4483}$ is noted $\propto$. Similar plots for the other storms of Figure (4) would yield parallel lines of good fit. The intercepts A of these lines on the i-axis at (D + γ) = 1, therefore theseintercepts will be plotted as straight lines on double logarithmic paper against the recurrence interval T. Hence for [A] = log $\propto$ + β [logT], find the magnitudes $\propto$ = 1.0151 and β = 0.3087to complete the numerical evaluation of the coefficients and with themin the equation.

$$i = \frac{\propto T^{\beta}}{(D+\gamma)^{\tau}} = \frac{1.0151 T^{0.3087}}{(D+0.5556)^{0.4483}} \qquad (4)$$

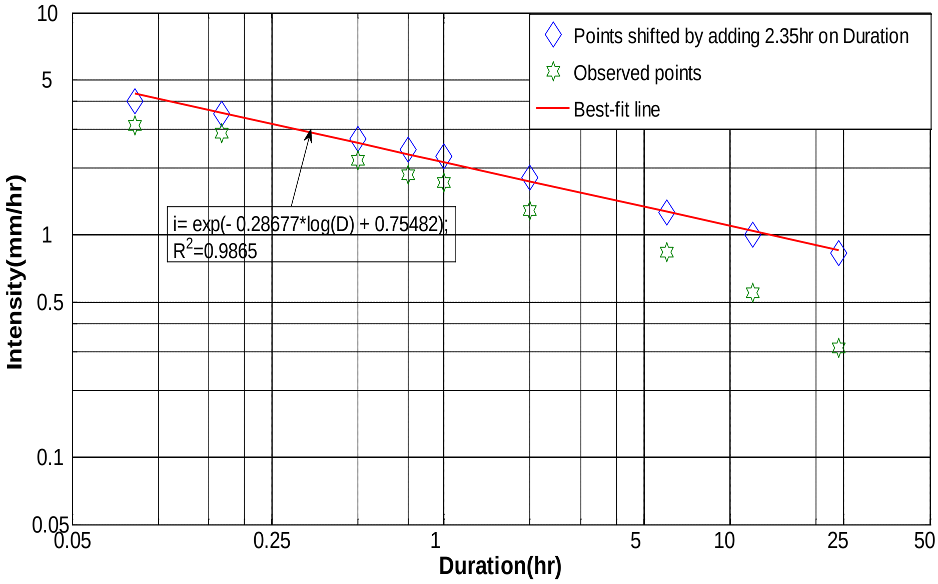

Fig. (4): Intensity-duration of 10-year rainstorm at Kerbala city.

2. Least-squares fitting

The numerical evaluation of the coefficients of IDF, calculated based on the regression analysis, would be obtained by applying a statistical software package called "DataFit" version 9.0.59 to produce four different equations of IDF as shown in Figures (5A,5B,5C and 5D). When certain measures are used such as standard error of anestimate and coefficient of multiple determination (R^2), the smallest values of this measure lead to the best fit. A brief overview of the equations of IDF is provided in the following subsections.

2.1.General equation of IDF

$$I=0.5915+01298\ln D+0.5885\ln T+0.0061\ln D^2+0.1066\ln T^2+0.1916\ln D\ln T+0.0047\ln D^3 +0.0250\ln T^3+0.0090\ln D\ln T^2+0.0018\ln D^2\ln T$$

Standard Error of the Estimate = 0.1327 and R^2= 0.9916

2.2.Ordinary equation of IDF

$$i = \frac{0.95165T^{0.312}}{(0.412+D)^{0.4543}} \quad (6)$$

Standard Error of the Estimate = 0.1876 and R^2=0.9809

2.3. Better equation of IDF

$$i = \frac{0.96905T^{0.31203}}{(0.42096+1.02165D)^{0.45434}} \quad (7)$$

Standard Error of the Estimate = 0.1871 and R^2=0.9810

2.4. The best equation of IDF

$$i = \frac{1.60368T^{0.31231}}{(1.08947+0.23547D^{0.23024})^{4.24411}} \quad (8)$$

Standard Error of the Estimate = 0.1871 and R^2=0.9843

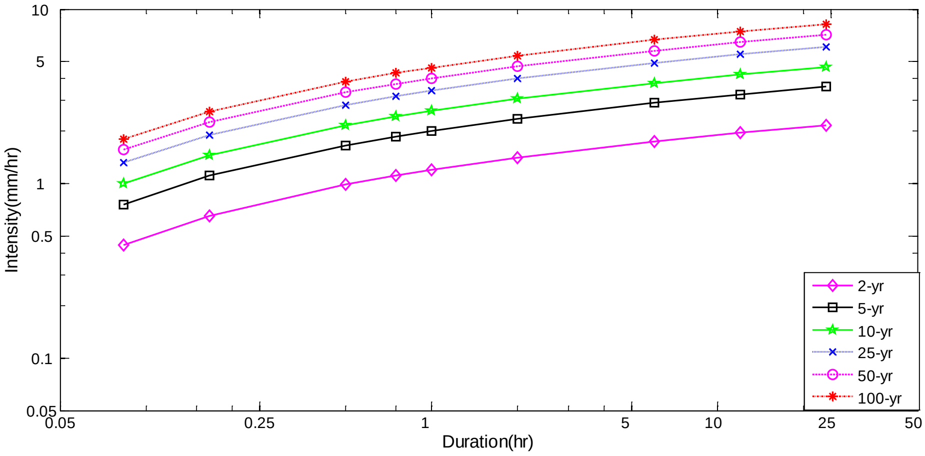

Fig. (5A): General equation of IDFat Kerbala city.

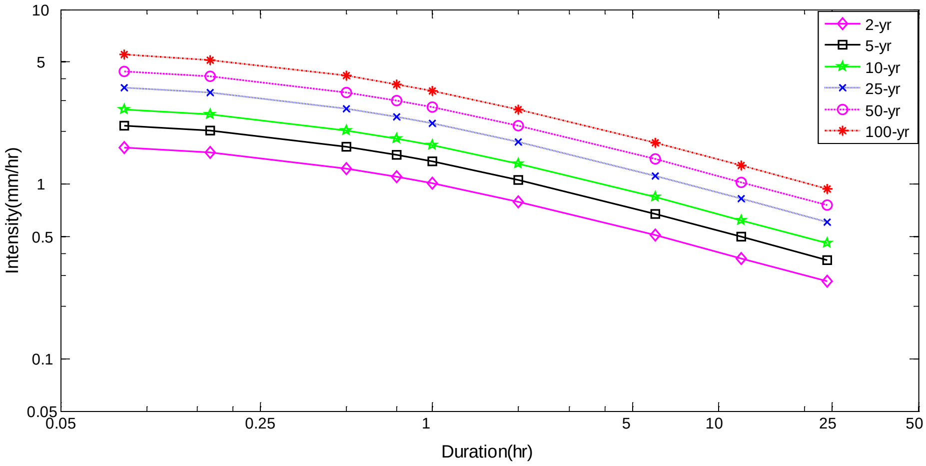

Fig. (5B): Ordinary equation of IDFat Kerbala city.

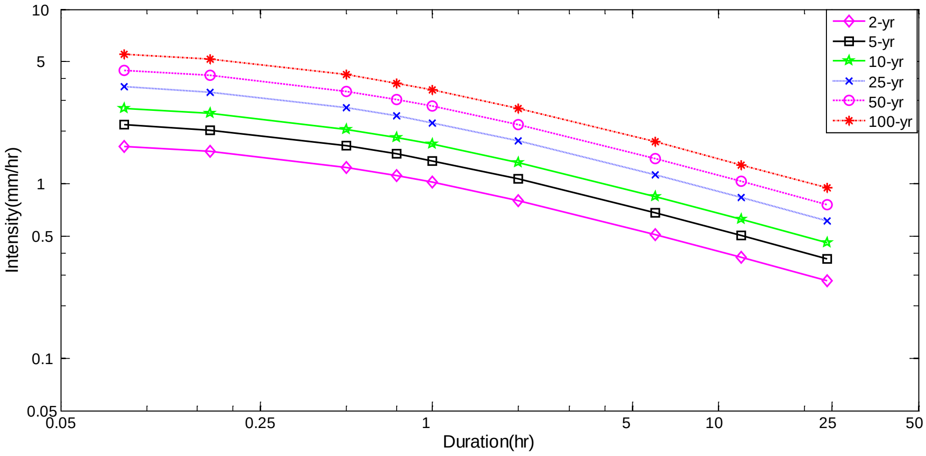

Fig. (5C): Better equation of IDFat Kerbala city.

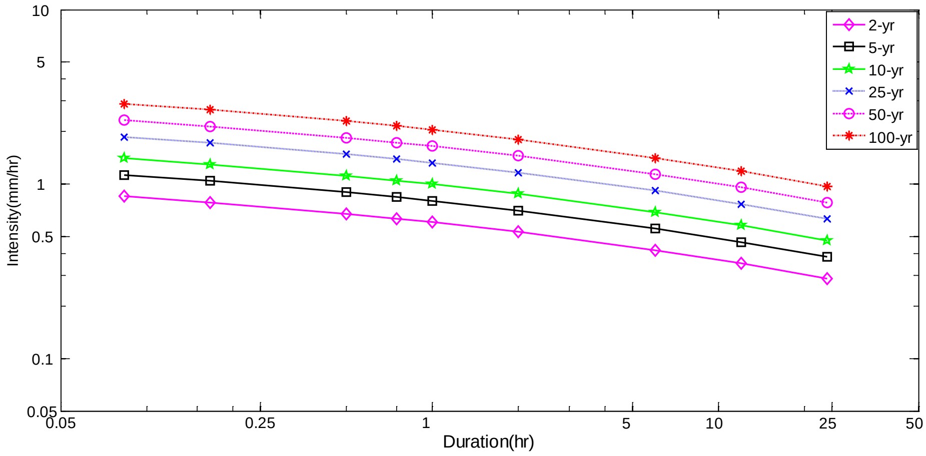

Fig. (5D): The best equation of IDFat Kerbala city.

3. Mathematical method

The formulas contained in the appendix A are applied to determine the parameters of an empirical IDF equation which is used to represent intensity-duration-frequency relationship.

$$i = \frac{1.0151T^{0.3087}}{(D+0.5556)^{0.4483}} \qquad (9)$$

Standard Error of the Estimate = 0.1884 and R^2=0.9839

4. Frequency analysis techniques

Two common frequency analysis techniques were used to develop the IDF equation. These techniques are: Gumbel distribution and Log Pearson type III (LPT III) distribution (Ibrahim H. E. , 2012).

Using the frequency analysis and frequency factor, the maximum intensity ofthe rainfall was estimated for different durations (10, 20, 30, 60, 120, 180, 360, 720 and 1440 min) and return periods (2, 5, 10, 25, 50 and100 years) using Gumbel andLPT III methods. The results are presented in tables 5 and 6 in both methods.The resulted two equations can be written as follows:

$$i = \frac{0.7106\,T^{0.5832}}{D^{0.3501}} \qquad (10)$$

$$i = \frac{0.5261\,T^{0.7332}}{D} \qquad (11)$$

Intensity is then plotted against duration as a function of return period on a log–log paper. The results shown in graphs as IDF curves in Figures 6 and 7 which are plotted using the Matlab program.

Table (5): Intensity (mm/hr) for different duration and frequency using Gumbel method for Kerbalacity.

Duration	Frequency					
hr	2	5	10	25	50	100
1/12	18.3569	31.8416	54.8263	54.8263	82.2332	123.5971
1/6	9.1730	15.9113	27.3967	27.3967	41.0919	61.7615
1/2	3.0583	5.3048	9.1341	9.1341	13.7001	20.5913
3/4	2.0388	3.5365	6.0894	6.0894	9.1334	13.7275
1	1.5291	2.6524	4.5670	4.5670	6.8500	10.2956
2	0.7646	1.3262	2.2835	2.2835	3.4250	5.1478
6	0.2549	0.4421	0.7612	0.7612	1.1417	1.7159
12	0.1274	0.2210	0.3806	0.3806	0.5708	0.8580
24	0.0637	0.1105	0.1903	0.1903	0.2854	0.4290

Table (6): Intensity (mm/hr) for different duration and frequency using LPT III method for Kerbala city.

Duration	Frequency					
hr	2	5	10	25	50	100
1/12	17.3828	41.5423	78.6961	125.3231	192.1115	349.3901
1/6	8.6862	20.7587	39.3244	62.6239	95.9982	174.5902
1/2	2.8960	6.9209	13.1108	20.8788	32.0058	58.2084
3/4	1.9307	4.6140	8.7405	13.9192	21.3372	38.8056
1	1.4480	3.4605	6.5554	10.4394	16.0029	29.1042
2	0.7240	1.7302	3.2777	5.2197	8.0014	14.5521
6	0.2413	0.5767	1.0926	1.7399	2.6671	4.8507
12	0.1207	0.2884	0.5463	0.8700	1.3336	2.4253
24	0.0603	0.1442	0.2731	0.4350	0.6668	1.2127

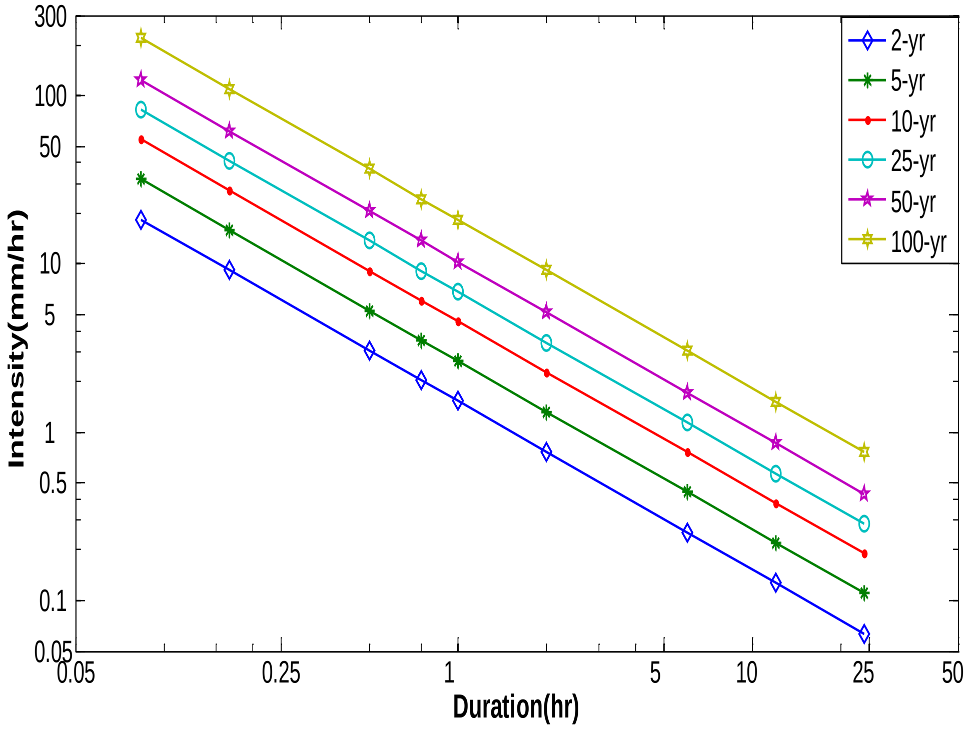

Fig. (6): IDF curves by Gumbel-Method at Kerbala city

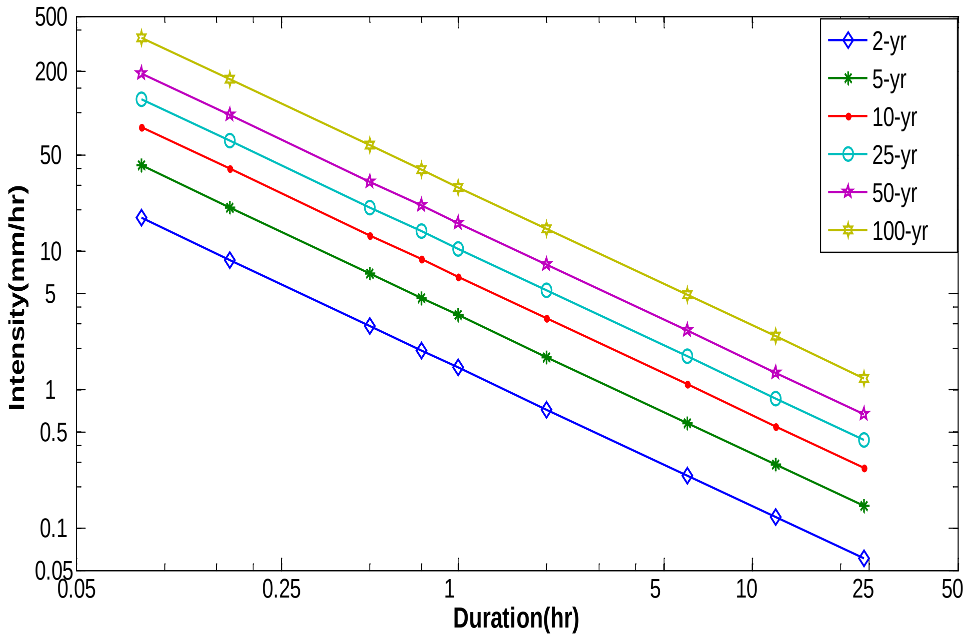

Fig. (7): IDF curves by LPT III-Method at Kerbala city

Tests of goodness of fit

The software(EasyFit) was used to perform the tests of goodness of fit by using Kolmogorov-Smirnov,Anderson-Darling, and Chi-Squared. The results are given in Table (7).Adoption of the Table (7) that are concluded the best method is the Log Pearson type III method depending on the Kolmogorov-Smirnov test and that the best equation of IDF method is better by Anderson-Darling test while the Mathematical method is the best way of using Chi-Squared test.

Table (7): Kolmogorov-Smirnov,Anderson-Darling, and Chi-Squared of maximum rainfall intensity data at Kerbala station.

Type	Kolmogorov-Smirnov	Anderson-Darling	Chi-Squared
Graphical fitting	0.06086	0.40886	0.43815
General equation of IDF	0.07111	0.29382	0.53887
Ordinary equation of IDF	0.06494	0.13437	0.55456
Better equation of IDF	0.05313	0.27032	0.60229
The best equation of IDF	0.0614	0.11999	0.75983
Mathematical method	0.05356	0.1774	0.40485
Gumbel Method	0.04764	0.38763	0.41878
Log Pearson type III	0.04512	0.13487	0.85463

Conclusion

The rainfall intensity-frequency relationship is one of the most commonly used tools in Water Resources Engineering, either for planning, designing and operation of water resources projects, or for various engineering projects against floods. It is therefore, important in determination of rainfall intensity for any desired duration and frequency as a guide in the design of water related structures. The availability of Rainfall Intensity-Duration-Frequency regimes will really make the design of some hydraulic structures easy for civil and water resources engineers, as well as other environmentalists carrying out works relating to rainfall around the study area. For Kerbala city, the study has been conducted to the formulation and construction of IDF curves using past available records of rainfall events by using empirical equation to represent Intensity-Duration-Frequency relationship for Kerbala city.
Using the tests of goodness of fit revealed that forKerbala city best method is the Log Pearson type III depending on the Kolmogorov-Smirnov test and the best equation for IDF method is obtaied by Anderson-Darling test, while for Mathematical method, the best way of is to use theChi-Squared test.Therefore, it isrecommendedto use these methods in estimating parameters of intensity-duration-frequency relationships. The proposed relationships of intensity-duration-frequency may be used in Iraq for design practices.

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Appendix A

This method (graphical fitting) does not exist in clear explanation in the references that explain it, therefore it will be explained here. The three variables, frequency, intensity and duration, are all related to each other. The data are normally presented as curves displaying two of the variables, such as intensity and duration, for a range of frequencies. These data are then used as the input in most storm water design processes. Assume that the curve is passing overthree points such as (i_1,t_1,T), (i_2,t_2,T), and (i_3,t_3,T), such that$i_2 = \sqrt{i_1 i_3}$,and substitution of these three points in the equation (5), yields:

$$i_1 = \frac{\mathrm{A}}{(\mathrm{D}_1+\gamma)^{\tau}} \;, i_2 = \frac{\mathrm{A}}{(\mathrm{D}_2+\gamma)^{\tau}} \;, i_3 = \frac{\mathrm{A}}{(\mathrm{D}_3+\gamma)^{\tau}} \qquad \text{(A1)}$$

Ratio of i_1/i_2 and i_3/i_2 , yields:

$$\frac{i_1}{i_2} = \frac{(\mathrm{D}_2+\gamma)^{\tau}}{(\mathrm{D}_1+\gamma)^{\tau}} \;, \qquad \text{and} \qquad \frac{i_3}{i_2} = \frac{(\mathrm{D}_2+\gamma)^{\tau}}{(\mathrm{D}_3+\gamma)^{\tau}} \qquad \text{(A2)}$$

$$\frac{i_1 i_3}{{i_2}^2} = \frac{(\mathrm{D}_2+\gamma)^{2\tau}}{(\mathrm{D}_1+\gamma)^{\tau}(\mathrm{D}_3+\gamma)^{\tau}} = 1 \qquad \text{(A3)}$$

Whence

$$d = \frac{D_2^2 - D_1 D_3}{D_3 + D_1 - 2D_2} \qquad \text{(A4)}$$

Assume Y=log i , A1=log $\propto$, B_1=X_1 , X_1=log T, B_2=- τ , X_2=log(D+ γ)
$Y = A1 + B_1 X_1 + B_2 X_2$ (A5)

$$\begin{pmatrix} \sum Y = A1N + B_1 \sum X_1 + B_2 \sum X_2 \\ \sum X_1 Y = A1 \sum X_1 + B_1 \sum X_1^2 + B_2 \sum X_1 X_2 \\ \sum X_2 Y = A1 \sum X_2 + B_1 \sum X_1 X_2 + B_2 \sum X_2^2 \end{pmatrix} \text{Solved by Grammer's Rule} \qquad \text{(A6)}$$

$$\left\{\begin{array}{l} A = \dfrac{\begin{vmatrix} \sum Y & \sum X_1 & \sum X_2 \\ \sum X_1 Y & \sum X_1^2 & \sum X_1 X_2 \\ \sum X_2 Y & \sum X_1 X_2 & \sum X_2^2 \end{vmatrix}}{\begin{vmatrix} N & \sum X_1 & \sum X_2 \\ \sum X_1 & \sum X_1^2 & \sum X_1 X_2 \\ \sum X_2 & \sum X_1 X_2 & \sum X_2^2 \end{vmatrix}} \rightarrow \propto = 10^A \; ; \qquad B_1 = \dfrac{\begin{vmatrix} N & \sum X_1 & \sum X_2 \\ \sum X_1 & \sum X_1 Y & \sum X_1 X_2 \\ \sum X_2 & \sum X_2 Y & \sum X_2^2 \end{vmatrix}}{\begin{vmatrix} N & \sum X_1 & \sum X_2 \\ \sum X_1 & \sum X_1^2 & \sum X_1 X_2 \\ \sum X_2 & \sum X_1 X_2 & \sum X_2^2 \end{vmatrix}} \rightarrow \mathrm{X} = B_1 \; ; \\ B_2 = \dfrac{\begin{vmatrix} N & \sum X_1 & \sum Y \\ \sum X_1 & \sum X_1^2 & \sum X_1 Y \\ \sum X_2 & \sum X_1 X_2 & \sum X_2 Y \end{vmatrix}}{\begin{vmatrix} N & \sum X_1 & \sum X_2 \\ \sum X_1 & \sum X_1^2 & \sum X_1 X_2 \\ \sum X_2 & \sum X_1 X_2 & \sum X_2^2 \end{vmatrix}} \rightarrow \tau = -B_2 \end{array}\right\} \quad (A7)$$