NUMERICAL STUDY TO EFFECT OF THERMAL LOADING ON J-INTEGRAL USING FINITE ELEMENT METHOD

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Abstract

In this work thermal loading is introduced to the J-Integral calculation using FEM. Study the effect of thermal loadings include temperature difference, convection heat transfer coefficient, temperature of fluid, and thermal conductivity on stresses values and J-integral . three Programs have been developed , first program is to evaluate temperature distributions in (conduction, convection), while the second program is to compute thermal loading stresses and another loading conditions, third program is to evaluate the J-Integral of opening mode (Mode I) under effect of thermal and mechanical boundary conditions is introduced. so the comparison of the result depends on the validation of the programs developed and the Extrapolation Methods.

Keywords : J-integral , finite element , stress , displacement

دراسة عددية لتأثير الحمل الحراري على تركيب نوع J باستخدام طريقة العناصر المحددة

احمد رزاق حسن المعهد التقني / السماوة

الخلاصــة

لقد تم في هذه الدراسة إضافة الحمل الحراري إلى حسابات تكامل J باستخدام طريقة العناصر المحددة, وذلك ب دراسة تأثير الحمل الحراري الذي يتضمن الفرق ب درجات الحرارة ومعامل انتقال الحرارة بالحمل ودرجة حرارة المائع والموصلية الحرارية على قيمة الاجهادات وقيمة تكامل J, حيث تم تطوير ثلاثة برامج, الاول لحساب توزيع درجات الحرارة بالحمل والتوصيل, والثاني لحساب الاجهادات الناتجة من تأثيرات الحمل الحراري والميكانيكي, واالثالث هو برنامج طريقة تكامل J (J- Integral program) لحساب قيمة J-Integral في حالة النمط المفتوح (Mode I) تحت تأثير الأحمال الحرارية والميكانيكية لأنواع مختلفة من المعادن. لقد أظهرت النتائج من البرنامج توافق جيد مع النتائج المستحصلة من طرق الاستقراء

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الرياضي (Extrapolation Methods) وبعض الطرق التحليلية.

NOMENCLATURE

Symbol	Description	unit			
a	The length of the crack	m	Δ	Difference between outer and inner surfaces	…….
E	Modulus of elasticity	pa	α	Coefficient of thermal expansion	$1/C^{\circ}$
h	Heat transfer coefficient of liquid layer	$W/m^2.^oC$	γ	Shear strain	…….
J	J-Integral	N/m	Γ	Boundary of the domain	…….
K	Thermal conductivity	$W/m.\,^oC$	ε	Normal strain	…….
l, m	Direction cosines of the normal vector	…….	θ	Angle	rad
N_i :	Shape function	…….	ν	Poisson's ratio	…….
n	Number of nodes	…….	ζ, η	Intrinsic coordinates variables	…….
N_e	Number of elements	…….	σ	Normal stress	Pa
q	Heat flux on the surface	W/m^2	τ	Shear stress	Pa
r	Radial distance from crack tip	m	χ	Total potential energy	J
R_o	Outer radius of a cylinder	m	Ω	Surface of the Domain	…….
R_i	Inner radius of a cylinder	m	$\underline{K}$	Stiffness matrix	
T_∞	Temperature of the surrounding	°C	$\underline{F}$	Force vector	
t	Thickness of structure	m	$\underline{U}$	Displacement vector	
Δ**T**	Temperature difference	°C			
T_x, T_y	Traction component in the x and y-directions	pa			
U	Strain energy	J			
u, v	Displacements in the x and y-directions	m			

1-Introduction

The mechanical loading is not only factor considered in the design of a structural or structure component. However , environmental condition such as temperature or extensive exposure to irradiation can affect the component. In the event of an accidents and even in some cases during normal operation. The effect of thermal loading on pre-existent flaw has attract considerable attention in the judgment of the safety of structure, figure (1) shows failure of material under thermal loading. Many high-performance structure and high-accuracy instrument have to consider thermal effect as a critical factor. Thermal effects on structures can be grouped into the following three principal categories [1]:

1. Changes in mechanical properties of materials (elastic modulus, fracture toughness and yield strength)
2. Creep phenomenon associated with time to hold,
3. Stresses arising from temperature change.

Figure (1-1) Shows failure via thermal cracking[2]

In the operation of gas turbine engines, thermal stresses can be as high as, or higher than, centrifugal stresses. The worst condition is the combinations of thermal, centrifugal and gas bending stresses at elevated temperatures result in high local stresses which can cause of cracking of turbine blades and rotor disks. Thus, thermal effects should not be ignored. The J-integral represents a way to calculate the strain energy release rate, or work (energy) per unit fracture surface area in a material. The theoretical concept of J-integral was developed in 1968 by Jim Rice[2] independently, he showed that an energetic contour path integral (called J) was independent of the path around a crack.

1.1 Aim Of Study

1.2 The aim of this work is to

1- Introduce the thermal loading to the J-Integral calculations .

2- Evaluate effects of temperature difference, convection heat transfer coefficient, temperature of fluid, and thermal conductivity on the stresses and J-Integral.

3- Three programs are to be constructed, two programs to evaluate the temperature and stresses distributions and a third program to compute the J-Integral under different type of loadings.

2-Literature Review

Sievers and Hofler (1986) [3] Numerical and experimental studied to investigate the applicability of the J-integral concept on mechanical and thermal shock with flaws in components of power plants during accidents They showed that for case 1a, J- integral increase with temperature increase and drop off with time increase and 'warm- prestressing - effect' is found, and for case 1b no 'warm- prestressing - effect' and the J-integral have distinctly higher values than in case 1a.

Lee and Park (1992) [4] Studied numerical analysis to drive expression of the J-integral for center cracked plate under thermal and mechanical loading conditions , They concluded that the increase of Specific Heat, Thermal Conductivity, Thermal Diffusivity, Young Modulus, the Temperature Difference of the crack side edge and crack length, increase the rate of J-integral under mode I thermal shock loading.

Ma and Liao (1996) [5] Used the thermal weight function method to determine the stress intensity factor for axial crack in hollow cylinders subjected to thermal shock and thermal loading , The result indicated that the maximum stress intensity factor may occur during the transient period and the overshot will be present especially for small crack.

Chen et al. (2006) [6] Used numerical analysis finite element method to determine stress intensity factor of multiple axial cracks in a coated hollow cylinder. The outer surface of cylinder is assumed insulated and the inner surface is exposed to convective cooling, tensile stress near the inner surface due to subsequent positive temperature gradient in radial direction. They obtained the stress intensity factor as a function of normalized quantities such as time, crack length, convection severity, and material constant.

3-Theoretical Analysis

In this part of study the effect of thermal loadings on suggested elements in fig .(3.1) and (3.2). The finite element method is a numerical method which can be used for the accurate solution of complex engineering problems. include temperature difference, convection heat transfer coefficient, temperature of fluid, and thermal conductivity on stresses values and J-integral is studies for **the following case study** :

1. Case Study 1 : J-Integral of Hollow Cylinder under Different Thermal Conduction and Pressurized Thermal Conduction with Radial Crack .
2. Case Study 2 : J-Integral of Hollow Cylinder under Thermal Convection and Pressurized Thermal Convection Loading and Variable Heat Transfer Coefficient with Radial Crack
3. Case Study 3 : J-Integral of Hollow Cylinder under Thermal Convection and Pressurized Thermal Convection Loading and Variable Temperature of Fluid with Radial Crack
4. Case Study 4 : J-Integral of Pressure Vessel under Thermal Convection and Pressurized Thermal Convection Loading and Variable Thermal conductivity with Radial Crack

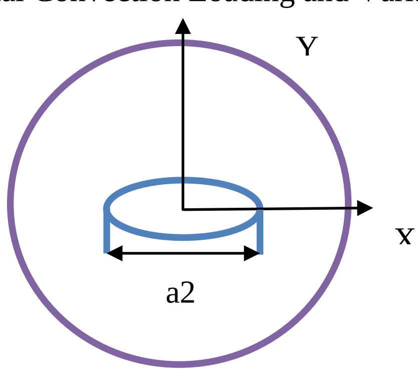

Figure (3.1) a Close domain with internal crack

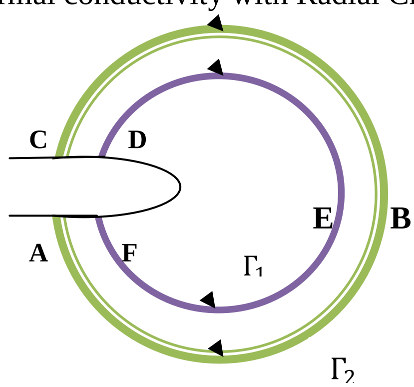

Figure (3.2) a Closed contour ABCDEF for a cracked body

3-1 Plane Stress

The assumption of plane stress is applicable for bodies whose dimension is very small in one of the coordinate direction. For this case the foregoing equations of elasticity theory are simplified to following[8]:

$$\sigma_z = \tau_{xz} = \tau_{yz} = 0 \qquad (1)$$

Hence,

$$\gamma_{xz} = \gamma_{yz} = 0 \qquad (2)$$

The stress and strain vectors component can be defined as follows:

$$\underline{\sigma} = \lfloor \sigma_x \ \sigma_y \ \tau_{xy} \rfloor \qquad (3)$$

$$\underline{\varepsilon} = \lfloor \varepsilon_x \ \varepsilon_y \ \gamma_{xy} \rfloor \qquad (4)$$

3-2 Plane Strain:

For bodies which are long (i.e. thick structure) and whose geometry and loading do not a very significantly in the longitudinal direction. It can be assumed that:

$$\varepsilon_z = \gamma_{xz} = \gamma_{yz} = 0 \qquad (5)$$

$$\sigma_z = \upsilon(\sigma_x + \sigma_y) \qquad (6)$$

3-3 Finite Element Formulation Of Linear Elastic 2D Problem:

The overall system of equations for the domain can be written as follows[8]:

$\underline{K}.\ \underline{U} = \underline{F}$ (7)

In order to solve the above system of equation the following boundary condition are applied:

1-at loaded nodes the displacement is unknown and the applied force is known.

2- at restrained nodes the load is unknown and the displacement is known.

After applying the above boundary condition the system of equations can be partitioned as follows:

$$\begin{bmatrix} \underline{K}_{uu} & \underline{K}_{up} \\ \underline{K}_{pu} & \underline{K}_{pp} \end{bmatrix} \begin{bmatrix} \underline{U}_{u} \\ \underline{U}_{p} \end{bmatrix} = \begin{bmatrix} \underline{F}_{u} \\ \underline{F}_{p} \end{bmatrix} \tag{8}$$

Where,

$\underline{U}_u$ = unknown displacement vector

$\underline{U}_p$ = prescribed displacement vector

$\underline{F}_u$ = prescribed force vector

$\underline{F}_p$ = unknown force vector

Hence, from the above equation, it can be shown that:

$$\underline{K}_{uu}\ \underline{U}_u + \underline{K}_{up}\ \underline{U}_p = \underline{F}_u$$

(9)

$$\underline{K}_{pu}\ \underline{U}_u + \underline{K}_{pp}\ \underline{U}_p = \underline{F}_p$$

I.e.

$$\underline{K}_{uu}\ \underline{U}_u = \underline{F}_u - \underline{K}_{up}\ \underline{U}_p \tag{10}$$

This represents a reduced system of equation.

3-4 Temperature Distribution And Thermal Stresses

At any finite element mesh, the temperature distribution **T** at any point (x,y) can be expressed as follows [9] :

$$\underline{T}(x,y) = \sum_{i=1}^{n} T_i\ N_i(x,y) \tag{11}$$

The overall system of equations of the domain can be written as :

$$[\ \underline{K}\ ]\ \ [\ \underline{T}\ ] = [\ \underline{F}\ ] \tag{12}$$

In general, when structure exposed to steady state linear temperature distribution **T**(x,y) an initial thermal strain will be generated which can be expressed as follows[10] :

$$\varepsilon_x^o = \varepsilon_y^o = \alpha^o \; T \tag{13}$$

$$\gamma_{xy} = 0 \tag{14}$$

Where,

for plane stress $\alpha^o = \alpha$

for plane strain $\alpha^o = (1+\upsilon)\alpha$

These initial strains will cause a thermal loading that can be represented as follows [18]:

$$F = \underline{F}_z^o + \underline{F}_\sigma^o \tag{15}$$

$$\underline{F}_z^o = \iint_{element} t \;\, \underline{B}^t \, \underline{D} \;\, \underline{\varepsilon}^o \;\, dx \;\, dy \tag{16}$$

$$\underline{F}_\sigma^o = -\iint_{element} t \;\, \underline{B}^t \, \underline{D} \;\, \underline{\sigma}^o \;\, dx \;\, dy \tag{17}$$

The above loading vector is defined as follows :

$\underline{F}_z^o$: is the loading vector equivalent to initial strain ε^o

$\underline{F}_\sigma^o$: is the loading vector equivalent to initial stress σ^o

3-5 J- Integral Formulation

Consider a two dimensional structure defined in terms of a domain Ω in the x-y plane and a thickness t in the z-direction [11].

Thus,

$$J = \iint_\Omega \frac{\partial w}{\partial x} \;\, dx \;\, dy - \int_\Gamma \underline{T}^t \, \frac{\partial U}{\partial x} ds \tag{18}$$

The first double integral in eq. (18) can be transformed to boundary integral by using green’s theorem.

$$J = \int_{\Gamma} w\ dy - \int_{\Gamma} \underline{T}^{t} \frac{\partial U}{\partial x} ds \tag{19}$$

Where,

$$W = \int_{0}^{z} \underline{\sigma}^{t} d\varepsilon$$

$$\underline{T} = \begin{bmatrix} T_x \\ T_y \end{bmatrix} = \begin{bmatrix} l\sigma_x + m\tau_{xy} \\ l\tau_{xy} + m\sigma_y \end{bmatrix}$$

For thermal loading, the strain energy term in the J-integral equation may be written as follows:

$$\int_{\Gamma} w\ dy = \int\int_{\Gamma} \underline{\sigma}^{t} \frac{d\varepsilon}{\partial a} dx\ dy$$

Where, $\underline{\varepsilon}$ represents the total strain vector. For the elastic stress strain relationship, it can be shown that:

$$\underline{\sigma} = \underline{D}(\underline{\varepsilon} - \underline{\varepsilon}^{o}) = \underline{D}\ \ \underline{\varepsilon}^{o}$$

Then, the J-integral expression for thermal loading only as:

$$J = \iint_{\Omega} \underline{\sigma}^{t} \frac{\partial \underline{\varepsilon}^{o}}{\partial x}\ dx\ dy - \int_{\Gamma} \underline{T}^{t} \frac{\partial U}{\partial x} ds \tag{20}$$

A numerical method Finite element method can be used to obtain an approximate solution, the accuracy of this solution depends on number of factors such as type of element used, the mesh representing, the geometry of the part . J-Integral programs that developed in this work is using to solve equation 20 . These programs depended on finite element method and coded using Fortran 90 language.

4-Results And Discussion

In order to investigate the effect of different type of thermal loading (conduction, convection, and combined pressure with conduction, and convection) in steady state without heat generation on J-Integral under different thermal and structural boundary conditions Several cases are taken for the analysis. The Crack tip coordinate system is shown in Fig.(4.1) , the boundary conditions and finite element model are shown in table (4.1) below. Due to symmetry of structure only one quarter of the cylinder has been modeled as shown in Fig.(4.2) .

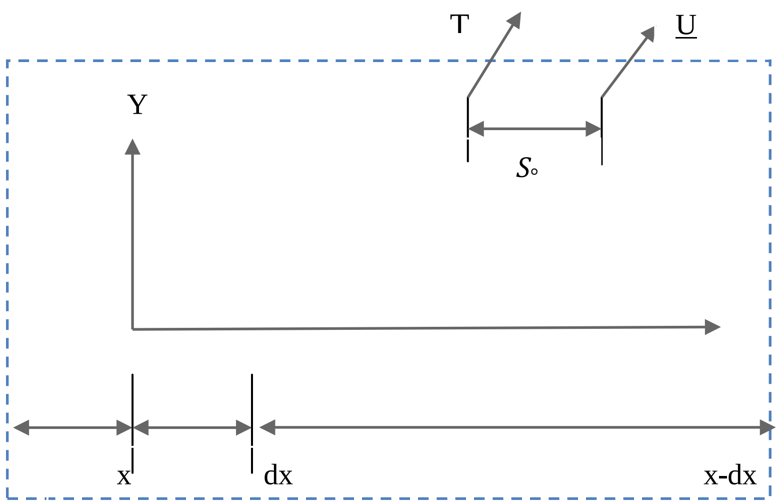

Figure (4.1) Crack tip coordinate system

Table (4.1) : shows the boundary condition of all cases

Young modulus	Poison ratio	Inner radius	Outer radius	Inner temperature	Outer temperature	Crack length
210	0.3	80 mm	100mm	-100	0	10
200	0.3	1.9	2.1	27	287	0.1

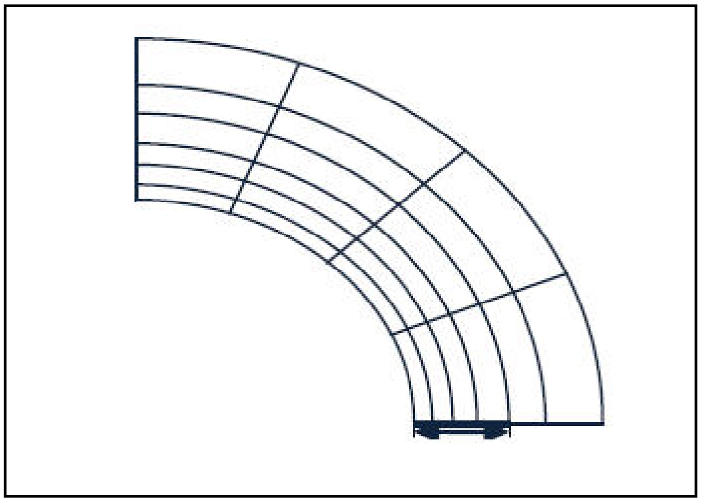

Fig . (4.2) . Quarter of cylinder has been modeled

Case Study 1: J-Integral of Hollow Cylinder under Different Thermal Conduction with Radial Crack

In this case hollow cylinder is exposed to thermal and pressurized thermal loading with radial crack are studied, the cylinder first exposed to different thermal conduction loading. For this case the J-integral for different crack ratios are plotted in Fig. (1), and the J-Integral with difference thermal boundary conditions and at crack length ratio 0.5 are shown in Fig. (2). which obtained from J-integral and extrapolation methods. From this figures, it can be observed that the value of the J-integral is increased resulting from the increasing of the stress intensity factor when the crack length ratio increase and higher temperature difference exists between the crack tip region and the outer surface of the cylinder, this is giving rise to a larger tensile stresses across the uncracked ligament.

Case Study 2: J-Integral of Hollow Cylinder under Thermal Convection and Variable Heat Transfer Coefficient with Radial Crack

In this case hollow cylinder subjected to thermal convection and pressurized thermal convection loading with radial crack are studied. The cylinder first exposed to different thermal convection loading. For this cases the J-integral for different crack ratios are plotted in Fig. (3), and the J-Integral with different convection heat transfer coefficient and at crack length ratio 0.5 are shown in Fig. (4).which obtained from J-integral and extrapolation methods, from this figures, it can be observed that a slight increase in the J-Integral when crack length ratio and heat transfer coefficient increased because of the proportional increasing of stress intensity factor as the crack length ratio increase and a slight increase in temperature difference exists between the crack tip region and the outer surface of the cylinder, giving a slight rise in tensile stresses across the uncracked ligament.

Case Study 3: J-Integral of Hollow Cylinder under Thermal Convection and Variable Temperature of Fluid with Radial Crack

In this case hollow cylinder subjected to thermal convection and pressurized thermal convection loading with radial crack are studied. The cylinder first exposed to different thermal convection loading. For this cases the J-integral for different crack ratios are plotted in Fig. (5), and the J-Integral with different temperature of fluid and at crack length ratio 0.5 are shown in fig. (6). which obtained from J-integral and extrapolation methods, from this figures, it can be observed that the value of the J-integral decreased when crack length ratio and temperature of the ambient fluid increase, because of the heat flux decreased and temperature distribution between the outer and inner surfaces of the cylinder also decreased, while lower tensile stresses across the uncracked ligament are observed.

Case Study 4: J-Integral of Pressure Vessel under Variable Thermal conductivity with Radial Crack

In this case hollow cylinder subjected to thermal convection and pressurized thermal convection loading with radial crack are studied. The cylinder first exposed to thermal convection loading, where thermal conductivity is changed. For this case the J-integral for different crack ratios are plotted in Fig. (7), and the J-Integral with different thermal conductivity and at crack length ratio 0.5 are shown in fig. (8). which obtained from J-integral and extrapolation methods, from this figures, it can be observed that the minimum influence of thermal conductivity on the J-Integral, where the increasing of the thermal conductivity leads to decrease the J-Integral, because of the decrease in a stress intensity factor as a result of decreasing the tensile stresses when the temperature difference (ΔT) decrease according to the Fourier's law.

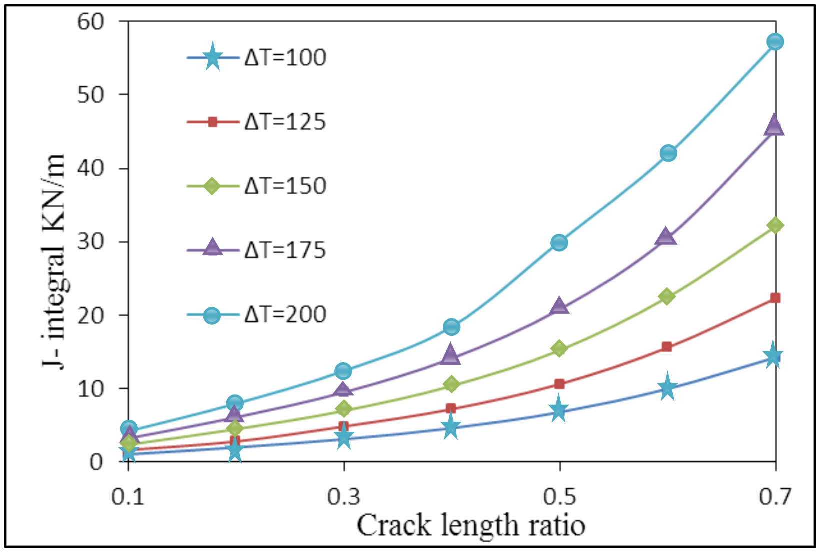

Fig. (1) J-integral for different crack ratio for case (1)

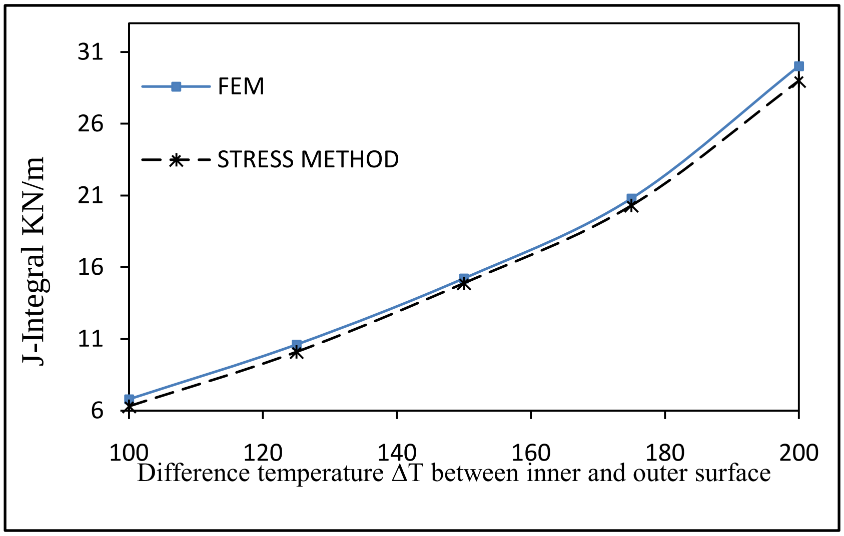

Fig. (2) Variation J-Integral with different temperature at crack length ratio 0.5 for Case 1

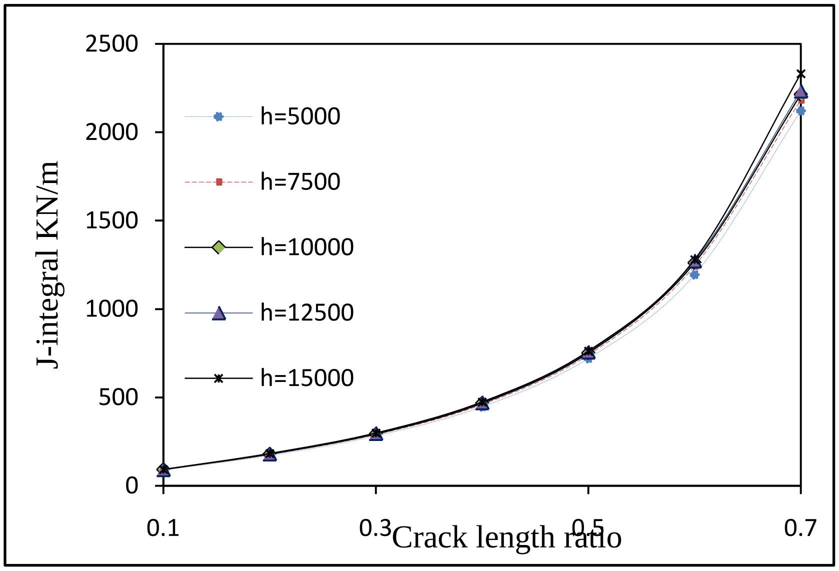

Fig. (3) J-integral for different crack length ratio for case 2

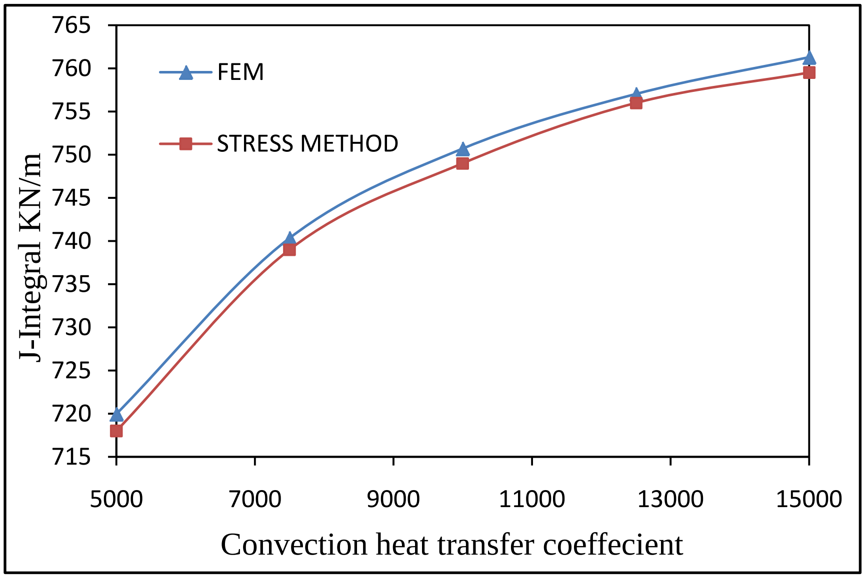

Fig. (4) Variation J-Integral with convection heat transfer coefficient at crack length ratio 0.5 for case 2

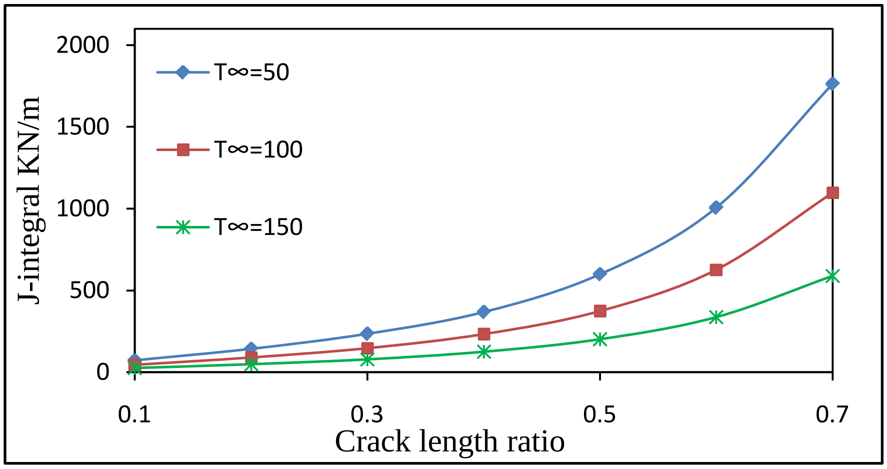

Fig. (5) J-integral for different crack length ratio for case 2

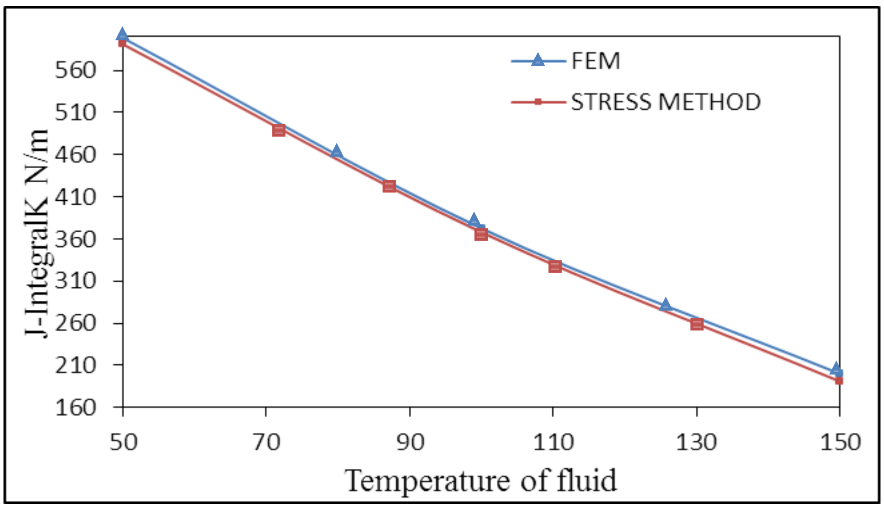

Fig. (6) Variation J-Integral with temperature of fluid at crack length ratio 0.5 for case (3a)

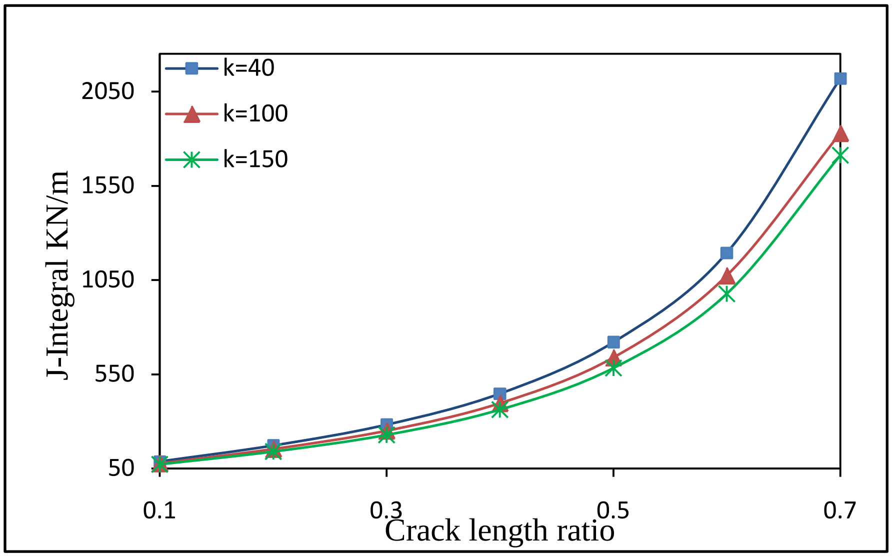

Fig. (7) J-integral for different crack length ratio for case 2

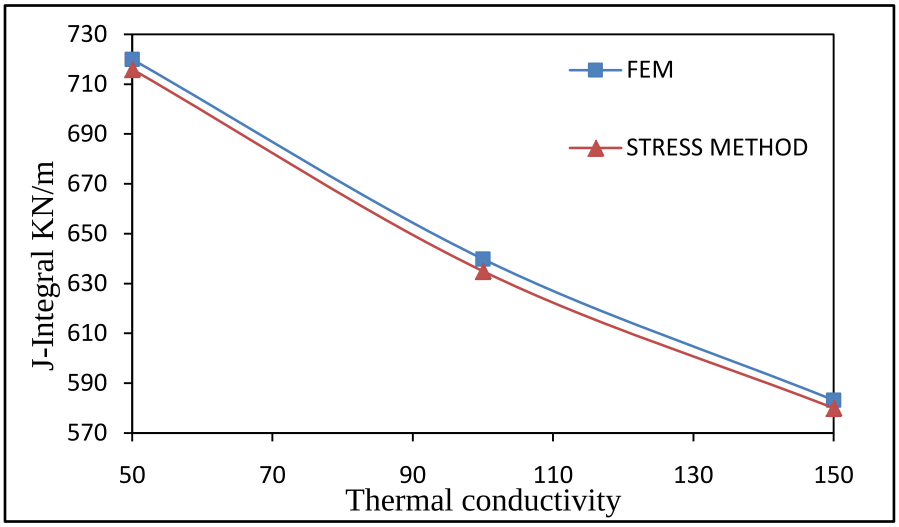

Fig. (8)Variation J-Integral with thermal conductivity at crack ratio 0.5 for case (4)

5- Conclusions

From the previous analysis, the following remarks can be concluded:

1- Conduction heat transfer has a great effect on stresses and J-integral, where the J-integral and stresses increase when the temperature difference in conduction increase
2- A slight effect on J-integral and stress has observe due to convection heat transfer coefficient, where J-integral and stresses are slightly increase when heat transfer coefficient increase.
3- Change in temperature of fluid has a noticeable effect on stresses and J-integral values, where increase of the temperature of fluid cause the J-integral and stresses to decrease.
4- The thermal conductivity exerted only a minimal influence on the J-Integral, where thermal conductivity has decrease slightly the J-Integral for each crack length.

6- Recommendation

The following recommendations are made for future work:
1-The work can be extended to deals with elastic-plastic analysis and three dimensional analyses.

2- The work can be developed to deals with unsteady state heat transfer, and internal heat generation, and radiation.

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