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Large deformation theory of thin steel cantilever beams under free end load cases

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Abstract

In this research, the large deflection of a cantilever tested steel beam of linear elastic material under the action of an external vertical concentrated force at the free end was numerically investigated. The definition of large deflection behavior indicates the inherent nonlinearity found in the response analysis in these beam systems. The analysis pertains to the domain of geometric nonlinearity, often expressing the equilibrium equation in a deformed structure. Numerous authors have implemented various numerical methodologies to address these issues. This paper investigates Rang-kuta numerical techniques for the numerical simulation of the problem. To achieve this purpose, a cantilever beam of length 1 meter and an isotropic thin steel plate with a rectangular cross-section were used. Assuming the beam material is isotropic, with a modulus of elasticity E = 200 GPa and a Poisson's ratio equal zero. The performance of the tested beam was assessed considering deflection and deflection angle. A parametric study is also included to investigate the effect of cross section dimensions (width x height) of steel plate on the bending and the deflection value of tested beams. The assessments indicate that the proposed method can be widely applied to measure large deflections in thin steel plate materials under concentered load at the free end of cantilever beams.

Keywords: *Cantilever beams; Large deflection; Thin material; Steel plate; Analytical solution; Bending.*

1. Introduction

A beam is an important structural element which carries stresses applied perpendicular to its longitudinal axis, primarily by providing resistance to bending. The static analysis of a beam includes the assessment of deflection, slope, curvature, stresses, moments, and other characteristics produced in the beam under specified loading conditions [1]. Cantilever bending is commonly used to investigate bending collapse characteristics under large deflections. According to the analysis results, one can determine whether the beam meets the criteria for providing sufficient resistance to prevent failure under the applied loading conditions. Usually, static analysis takes place using linear models to simplify the analysis [2]. However, linear models are not able to capture the actual behaviour of a structure, as nearly all structures present nonlinear behaviour before they reach their limit of resistance. Thus, numerical results determined by these linear theories are unsuitable for large deflection predictions, as they may result in severe errors. Recent developments in mathematical mechanics have allowed researchers to better describe systems by capturing the nonlinear response to these structures. The Euler-Bernoulli beam model is suitable for predicting the mechanical behavior of slender beams.

Euler-Bernoulli beams have been the target of significant study about nonlinearity resulting from the geometry of the beam and the materials. Mathematical calculation of large deflections in Euler-Bernoulli beams has a long research history. For instance, Lewis and Monasa [3, 4] examined the large deflections of thin cantilever beams due to a concentrated force and an end moment at the free end, respectively. Bisshop and Druckerin [5] studied the large deflection of cantilever tested beams with variable cross section (rectangular and circular cross-sections). They primarily implemented the Runge-Kutta method and then used a predictor-corrector to gradually improve their results. Lee [6] studied the large deformation of cantilever beams constructed from Ludwick-type material under a combined loading comprising a uniformly distributed load and a vertical concentrated loading at the free end. That numerical solution was obtained through the application of Butcher's fifth-order Runge-Kutta method and is presented in tabular format. Additionally, Brojan et al. [7] estimated the major deflections of a non-prismatic slender cantilever beam that is subjected to a concentrated moment at its free end. The research developed an exact moment-curvature equation for materials that obey to the generalized Ludwick's law. This formula is suitable for analysing beams with variable loading and support conditions. The results of numerical examples obtained from our materially and geometrically nonlinear analysis clearly indicate complex nonlinear behaviour in the analysed cantilever beams. Kimiaeifar et al. [8] developed a homotopy semi-analytical solution for analyzing large deflections in a cantilever beam with free ends and uniformly distributed loads. For the purpose of comparison, deflections were estimated and compared with the results of the finite element method, which served as the reference. The results show that the proposed solution is very accurate, effective, and convenient for the addressed issue, as well as suitable for a wide range of practical problems. Similar findings were also observed in [9]. In another study by Borboni and De Santis [10] performed a comprehensive investigation of the large deflections of an asymmetric Ludwick cantilever beam subjected to a horizontal force, a vertical force, and a bending torque at the free end. The Euler-Bernoulli bending beam theory is used to study large deflections. This theory says that cross-sections stay flat and perpendicular to the neutral surface after deformation, and their shape and area don't change either. The mechanical model developed from previous hypotheses contains two types of non-linearity: the first due to material properties, and the second resulting from large deformations. The suggested method worked well for solving the nonlinear algebraic system and the nonlinear second-order ordinary differential equation, which made the problem smaller. Yuan et al. [11] evaluated the effective width of steel-concrete composite beams based on a specific beam section. For this purpose, this paper first presents the development of two theoretical models for composite beams. Validation of the theoretical models is performed through comparison of the theoretical predictions with the results obtained from more complex finite element simulations. The results show that the width of the concrete slab, the span of the beam, and the thickness of the floor slab primarily influence the effective width. Simplified design formulas for computing the effective width are proposed. Comparisons between the results of the simplified formulas and the test results indicate the accuracy of the proposed formulas. Nguyen et al. [12] introduce an analytical model for thin-walled open-section beams that utilize functionally graded materials (FGMs). The proposed theory considers restrained warping applicable to the thin-walled FG beam based on Vlasov's assumptions. Khosravi et al. [13] introduce a numerical method that simulates a cantilever beam, expressing it as a boundary value problem under mixed conditions. Two novel numerical techniques are investigated. The first is based on a spectral method utilising a modal Bernstein polynomial basis. Next, we implement the second-order convolution quadrature method to discretize the problem, incorporating a finite difference approximation for the Neumann boundary condition on the beam's free end. Comparison with the experimental data and the existing numerical and semi-analytical methods demonstrate the accuracy and efficiency of the proposed methods. Pandit et al. [14] present a class of problems involving space-constrained loading on thin beams with large deflections. Analytical solutions to such problems when the material is elastoplastic are difficult to obtain. In this paper, an incremental method is employed to solve the governing differential equation. Local elastic unloading, which may occur in large deflection problems, is naturally incorporated in the formulation. A non-dimensional parameter depending on both material and geometry is obtained here via the process of normalisation of the bending moment in two different ways. This parameter is seen to govern the fixed end moment versus end displacement response in an elasto-plastic case. Li et al. [15] propose a new steel-concrete composite cantilever beam. Six steel-concrete, double-sided composite cantilever beams were designed and experimentally tested and theoretically analysed to study their main mechanical properties. The results show that the bottom concrete slab of the double-sided composite cantilever beam can effectively enhance the stability of the bottom flange of the steel girder, the ultimate bearing capacity of the composite beam, and stiffness.

However, the previously mentioned research was limited to the static analysis of cantilever beams with variable cross-sections subjected to various loading conditions. The load conditions include distributed concentrated loads, moment loads, and combined loads at the free end. Generally, thin-walled beams built from anisotropic materials present complex and related structural behavior. Therefore, the design process must incorporate warping and other coupling effects, which may have practical significance. Additionally, commonly, just accuracy is evaluated, while calculation time and simplicity are neglected. On the other hand, further numerical research is needed in this area. The point of this study is to look into the spread of steel materials in thin-walled cantilever beams and come up with a complete analytical model based on variation formulation. This paper considers the rectangular sections that are subject to end point load at their free end. Additionally, derive and solve the governing equations to accurately determine the large deflection and twist angle.

2. Theoretical and numerical background

A typical illustration of deflections is given in Figure 1 for a cantilever subjected to perpendicular force at the free end, where P is the concentrated load. The vertical displacement is denoted by y, and the deflection angle is represented by θ. Furthermore, the arc length is denoted by ds, the horizontal displacement between M1 and M2 is denoted by x, and the slope in point M1 is θ, and the slope in point M2 is θ+dθ. The intersection of vertical lines with points (M1, M2) occurs through the center of curvature at point (o). This means that a deflected shape is part of a circle, with its center point (o) and radius (ρ).

ds = ρ dθ (by math)

Distance on curve equal to radius multiplied by confined angle (or) internal angle, see Figure 1.

$$\therefore \ \rho = \frac{ds}{d\theta} \tag{1}$$

Where ρ is radius, ds is arch length, and dθ is internal angle.

$$The\ Curvature \quad k = \frac{1}{\rho}\left[\frac{1}{Radius}\right]$$

$$\therefore \ \frac{1}{\rho} = \frac{d\theta}{ds} = k \tag{2}$$

Assume (θ) very smaller angle.

$\therefore dx = ds\,, \cos \backsimeq 1, \tan\theta \backsimeq \theta$

$\tan\theta = \frac{dy}{dx} \qquad [deflection]$

$\therefore \theta = \frac{dy}{dx}$ (3)

The basic equation is derived in the form of:

$\therefore \frac{d\theta}{ds} = \frac{d_2y}{dx^2}$ (4)

From equation (2) we can obtain

$\frac{1}{\rho} = \frac{d_2y}{dx^2}$ (5)

From flexural theory can be written as

$\frac{1}{\rho} = \frac{M}{EI}$ (6)

$\therefore \frac{M}{EI} = \frac{d_2y}{dx^2}$ (7)

$(pant) EI\ ragtied\ section$

$\therefore M = \frac{d_2y}{dx^2}\ EI$ (8)

$or\ write$:

$EI\ y'' = M\ (intreal\ moment)$ (9)

$If\ we\ used\ the\ exact\ value$

$for\ the\ part\ \left(\frac{1}{\rho}\right) from\ mathematics\ books$

$Then\ we\ get\ to\ this\ equation$

$\frac{1}{\rho} = \frac{\frac{d_2y}{dx^2}}{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}$ (10)

∴ Deflection equation

$\frac{M}{EI} = \frac{\frac{d_2y}{dx^2}}{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}$ (11)

Or used:

$\frac{M}{EI} = \frac{d_2y}{dx^2}$ (12)

$We\ can\ neglect\ \left(\frac{dy}{dx}\right) because\ it's\ very\ small\ value$

Deriving The maximum value to deflection by used engineering method. (exact).

Solution:

$EI\ y'' = \frac{d_2y}{dx^2}\ EI = M(x)A$ (13)

$\therefore \frac{d_2y}{dx^2}\ EI = M(x) intread\ moment$ (14)

From the vertical equilibrium of the cantilever beam, the moment value M at each support is determined as follows:

$\sum fy = 0.0\ (+)\ \uparrow$

$+Ay - P = 0.0 \Longrightarrow Ay = P\ \uparrow$ (15)

$\sum M\ at\ A = 0.0\ (+)$

$+P \times\ \alpha - MA = 0.0\ \Longrightarrow cw\ MA = PL \quad c.c.w$ (16)

$\sum M\ at\ x = 0.0\ (+)$

$+P \times x - PL - M(x) = c.w\ 0.0$ (see Figure 2) (17)

$\therefore M(x) = Px - PL$ (18)

$Intreal\ moment\ to\ long\ beam$

$EI\ y'' = M(x)$ (19)

$\int EI\ y'' = \int P\ (x - L)$

$\int EI\ y' = \int P\left[\frac{x^2}{2} - Lx + C_1\right] \qquad (\theta)\ rottam$ (20)

$EI\ y = P\left[\frac{x^3}{6} - \frac{Lx^2}{2} + C_1x + C_2\right] \qquad (\delta)\ deflection$ (21)

$Used\ geometric\ nature\ to\ fixed\ suppurt$

$at \quad x = 0.0 \quad , \quad \theta = 0.0 \qquad sub\ equation\ (20)$

$EI\ (0) = P\left[\frac{(0)^2}{2} - L(0) + C_1\right]$

$\therefore C_1 = 0.0$ sub $in\ equation\ (21)$

$EI\ y = P\left[\frac{x^3}{6} - \frac{Lx^2}{2} + C_2\right]$

$In\ Fixed\ suppart \quad at \quad x = 0.0 \quad , \quad y = 0.0$

$EI\ (0.0) = P\left[\frac{(0.0)^3}{6} - \frac{L(0.0)^2}{2} + C_2\right]$

$\therefore C_2 = 0.0$

$\therefore\ Equation\ exact\ deflection$

$$EI\ y = P\left[\frac{x^3}{6} - \frac{Lx^2}{2}\right] \tag{22}$$

$Max\ diflection\ in\ free\ end\ when\ \ x = \alpha$

$$EI\ y = P\ \left[\frac{L^3}{6} - \frac{L^3}{2}\right] \tag{23}$$

$$EI\ y = P\ \left[\frac{L^3 - 3L^3}{6}\right]$$

$$EI\ y = P\left[\frac{-2L^3}{6}\right]$$

$$EI\ y = P\left[\frac{-1L^3}{3}\right] \tag{24}$$

$$\therefore\ y = \frac{1}{EI}\left(\frac{-PL^3}{3}\right) \tag{25}$$

$$\therefore\ \ \delta = \left(\frac{-PL^3}{3EI}\right)\downarrow \tag{26}$$

$Exact\ deflection\ beam\ need\ compare\ it\ with\ numrieal\ solution.$

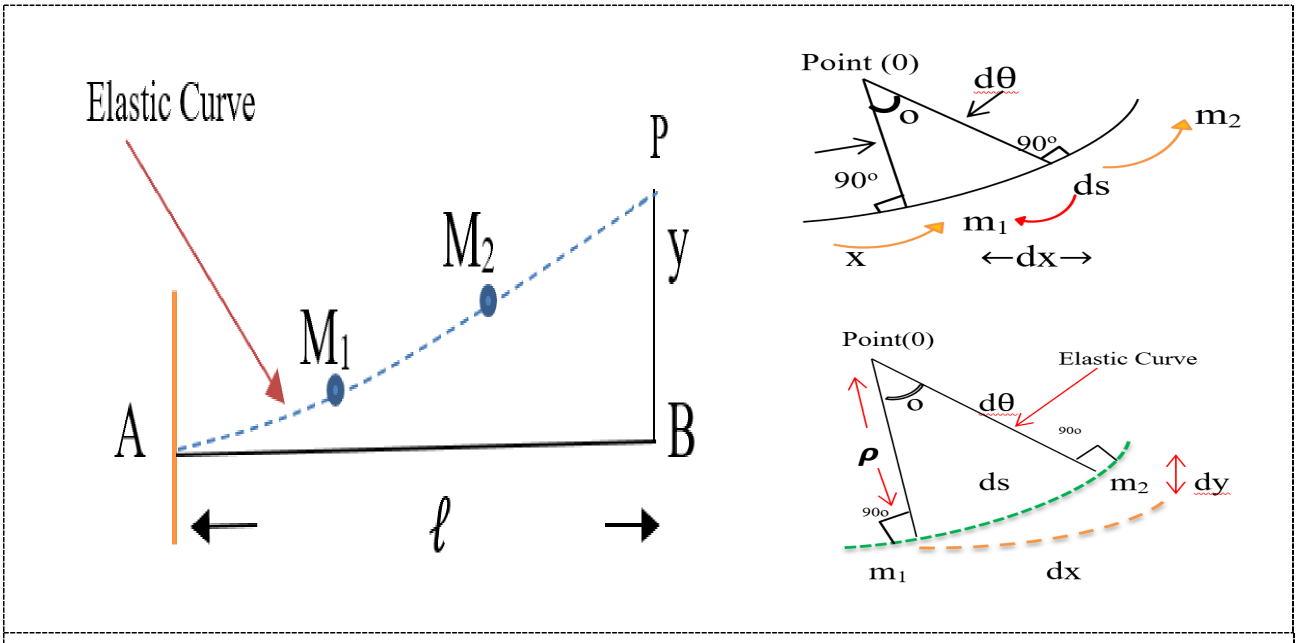


Fig. 1: Cantilever tested beam subjected to an external vertically concentrated force at the free end

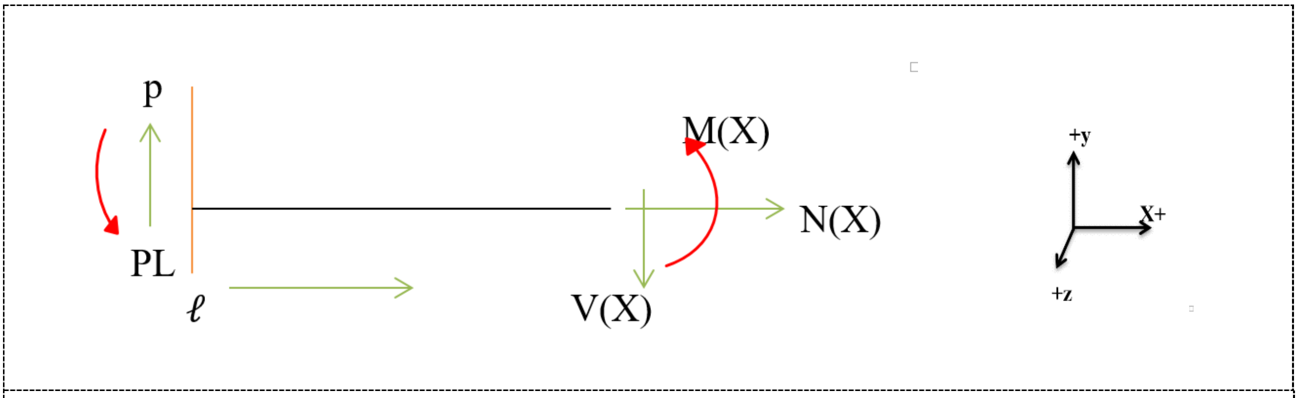


Fig. 2: Equilibrium of the deformed cantilever beam.

3. Mathematical formulation

The aim of nonlinear analysis is to explain different nonlinearities and the structure of basic numerical equations utilised to assess the nonlinear response of a structural system. To achieve this, we must establish the body of interest's equilibrium in its current configuration. The aim is to establish an approximate solution for large deflections of the cantilever beam using the general equations for bending stress beams. In cases with large deflections, the angle ϕ remains minimal for every location along the cantilever beam. The stresses generated on the section due to the force applied on the longitudinal axis of the beam are referred to as bending stress, and the beam itself is referred to as a flexural member.

$$f = \frac{M*y}{I_{sec}} \tag{27}$$

where f is bending stress, M is bending moments, y is the height top compression section, and I refer to the moment of inertia of the beam cross section about the neutral axis. For this study, the section most exposed to stress should take the maximum bending moment (see Figure 3), as indicated in the equation below.

$$f_{max} = \frac{MC}{I_{sec}} \tag{28}$$

where c is the highest distance for bending stress. Consequently, we can write the flexural formula below.

$$f_{max} = \frac{M_y C_x}{I_{x\,section}} \tag{29}$$

The basic formula is presented from Equations 2 and 3 as follows:

$$f_{max} = \frac{My}{S_x} \tag{30}$$

S_x stands for the elastic section modulus. In this study, we examine the elastic zone of steel beams, as depicted in Figure 4. The calculator displays the steel section's bending moment, which contributes to the deflection calculation in Figure 5. Figure 5 makes it easy to calculate the compression force by vertical equilibrium.

$$F_{top} = \frac{1}{2}\sigma_y \times \frac{d}{2} \times b \Longrightarrow \frac{\sigma_y bd}{4} \tag{31}$$

Figure 5 makes it easy to calculate the tension force by vertical equilibrium.

$$F_T = \frac{1}{2}\sigma_y \times \frac{d}{2} \times b \Longrightarrow \frac{\sigma_y bd}{4} \tag{32}$$

where the arm compression forces a form the top fibre compression is given by the equation (Figure 5).

$$a = \left(d - 2 \times \left(\frac{d}{6}\right)\right) \Longrightarrow \frac{d}{1} - \frac{d}{3} = \frac{2d}{3} \tag{33}$$

$$\therefore Moment\ at\ tenstion = \frac{\sigma_y bd}{4} + \frac{2d}{3} = \frac{\sigma_y bd^2}{6} \tag{34}$$

$$\therefore M_y = \sigma_y \times \frac{bd^2}{6} \tag{35}$$

The large deflection effects on the cantilever beam are investigated by numerical methods in the calculation. As a result, the Rang-kuta method is one of the most accurate numerical calculation methods.

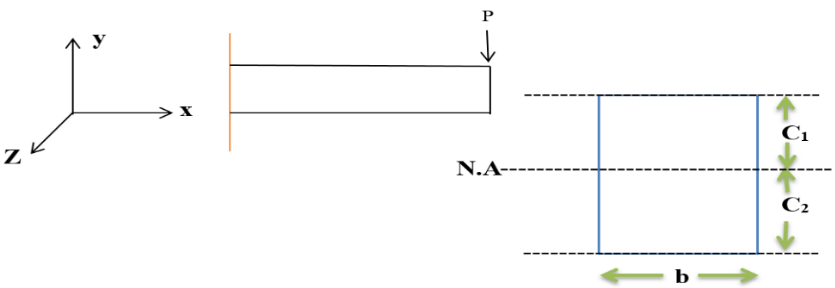


Fig. 3: Representation of cantilever beam and the corresponding maximum bending moment's behavior under axial loading.

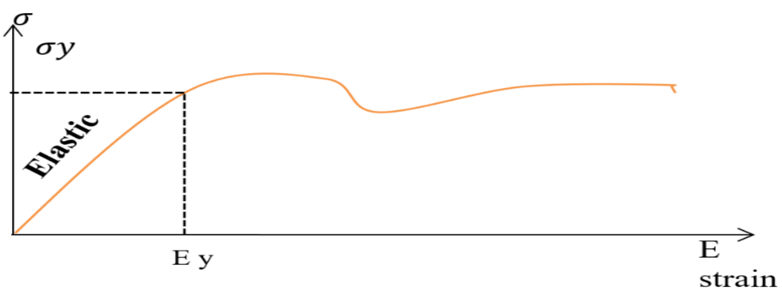


Fig. 4: Stress-strain diagram for cantilever beams [16].

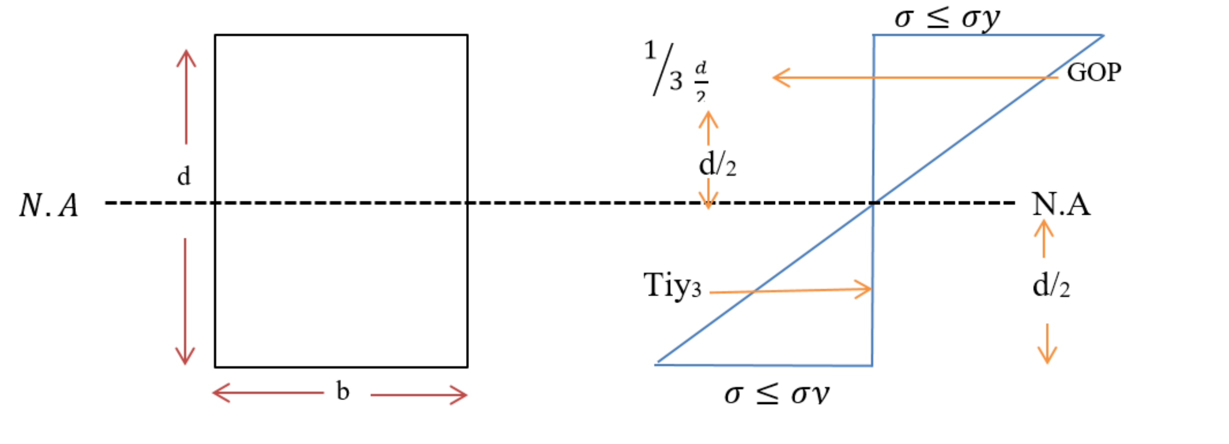


Fig. 5: Diagram for steel beam in elastic zone condition.

3.1 Analysis using rang-kuta method

A cantilever beam is considered where thin steel material properties and geometries are taken for large deflection. In the Rang-kuta method, the displacement fields follow the higher-order fourth-degree (4ed) deformation cantilever beam theories. The application of this method is seen below. The rectangular Cartesian coordinate system x, y, and z is aligned with the length, width, and height of the beam, respectively, as seen in the Figure 6. We take a cantilever beam as an example, measuring 1.0 m in length, 0.01 m in height, and 0.01 m in width. We assume that the beam material is isotropic with E = 200 GPa and that Poisson's ratio is zero. The stress placed on the cantilever beam (σ = 400 MPa), step size (h = 0.1), and section dimension (100 mm × 100 mm) are shown. Therefore, to calculate the deflection of any point along length beams using equation (35).

$$M_{yeilding} = \sigma_y \times \frac{bd^2}{6} \tag{35}$$

where My is the cantilever beam's bending moment support, equal to the distance between the concentrated load and the load value, and b and d are the beam's width and height.

$$P_y \times 1M = 4 \times 10^5 \frac{KN}{M^2} \times \frac{(0.1)(0.1)^2 M^3}{6} \tag{36}$$

$$\therefore P_y = 66.66KN$$

Thus, to stay in elastic Zone used $P < P_y$, $p = 65KN$ applied force to beam using equilibrium equations $, A_y = 65kN\ (up), and\quad M_A = 65\quad kN.m$ (C.C.W).

To determine the deflection of the equation, we must determine the internal moment at the given distance (x) (see Figure 7).

$$\therefore M_{(x)} = 65(x-1)$$

Substituting equations deflection (12) for E is the modulus of elasticity for steel beam equal to 200000 MPa, and I is the second moment of inertia of the cross section of the beam with respect to axis y equal to $\frac{bh^3}{12}$ we get.

$$EI\frac{d^2y}{dx^2} = M_{(x)} \tag{12}$$

$$200000 \times 10^3 \times \frac{(0.1)(0.1)^3}{12} \times \frac{d^2y}{dx^2} = 65(x-1)$$

$$\therefore \frac{d^2y}{dx^2} = \frac{1}{25.641}(x-1) \tag{37}$$

Used the numerical method to solve all details and assume:

$$Z = y' \quad , \quad z' = y''$$

This is a proposed non-linear deflection equation for cantilever steel beams subjected to concentrated load at the free end.

$$y_{n+1} = \frac{1}{6}[K_1 + 2(K_2 + K_3) + K_4] \tag{38}$$

Also, the proposed non-linear rottain equation for cantilever steel beams subjected to concentrated load at the free end.

$$Z_{n+1} = \frac{1}{6}\left[\sigma_1 + 2(\sigma_2 + \sigma_3) - \frac{+\sigma_4}{17} - \right] \tag{39}$$

In Table 1, the numerical results obtained by the Rang-kuta method are reported to obtain the real value for deflection cantilever beams, so we have.

$$\therefore y_{max} = 0.0128M \qquad or \simeq -13mm\ deflection$$

The validity of the results obtained from the Rang-kuta method is established by comparing them with the basic general equation for deflection cantilever beams.

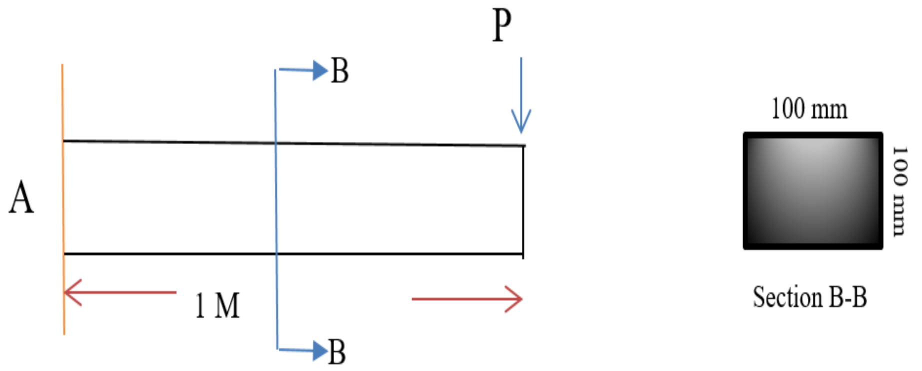
$$\Delta = \frac{PL^3}{3EI} = 13\ mm \quad \therefore 0.k$$

$$and\ \theta = 0.0196 \quad c.c.w$$

For more accuracy, this type of cantilever beam, shown below the relationship:

$$\frac{M}{EI} = \frac{\left(\frac{d^2y}{dx^2}\right)}{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}} \tag{40}$$

By repeating the same previous accounts, the results are tabulated in Table 2. It is seen that this equation (12) method gives better accuracy.



(a) (b)

Fig. 6: (a) Schematic illustrating the geometry of a cantilever beam and (b) the rectangular cross-section of the beam.

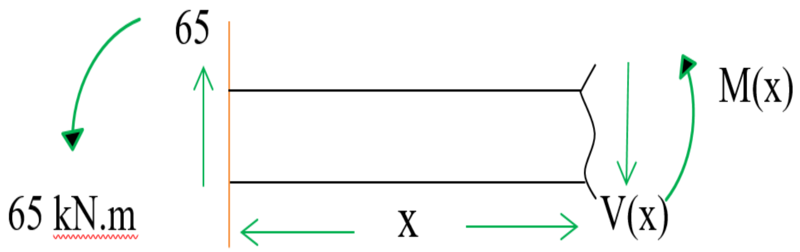


Fig. 7: Internal forces for cantilever beams at distance x.

Table 1: Numerical results for the rang-kuta method.

h	Deflection	Rotation Angle
0.1	-0.00019	-0.00371
0.2	-0.00073	-0.00703
0.3	-0.00158	-0.0103
0.4	-0.0027	-0.0128
0.5	-0.0040	-0.0150
0.6	-0.0055	-0.0168
0.7	-0.0073	-0.0181
0.8	-0.009	-0.0190
0.9	-0.011	-0.0195
1	-0.0128	-0.0196

Table 2: Numerical results obtained by equation deflection no. (12).

h	y_m	Rotation Angle θ
0.1	-0.00019	-0.00371
0.2	-0.0073	-0.00703
0.3	-0.001583	-0.009955
0.4	-0.002689	-0.01249
0.5	-0.004049	-0.014636
0.6	-0.005604	-0.017392
0.7	-0.007315	-0.017758
0.8	-0.009143	-0.018733
0.9	-0.011049	-0.019318
1	-0.012994	-0.019513

4. Numerical results

This section presents many numerical examples to compare the differences in large deflections of a non-linear cantilever steel beam. Additionally, an optimal design for a cantilever steel beam, based on a specified work hardening law, is shown.

It was studied in the case that the placed load less than loading that cause failure, that means staying in the elastic zone. To demonstrate the use of the beam theory, the Excel sheet is used to analyse three example cases.

4.1 Case 1

In the following numerical math calculations, we consider a cantilever beam with a length of L1 m and a rectangular cross-section of b by h, where b is 0.1 m and h is 0.01 m (see Figure 8). Therefore, numerical results are presented only for a cantilever steel beam with and without neglecting the rotation angle. Table 3 shows the results for numerical cantilever beams under static loads.

$$\text{Stress} = \frac{MC}{I}$$

$$\therefore 400 = \frac{(P*1)10^6*\emptyset}{\frac{100*103}{12}}$$

$\therefore P = 0.66N.(KN)$

Used $P_{Yielding}$=0.6∅ N.(KN)

The most significant fact that is seen from the numerical results (see Table 3) is that the yin length is 1 meter with angle rotation (θ) greater than Y in neglect angle rotation (θ).

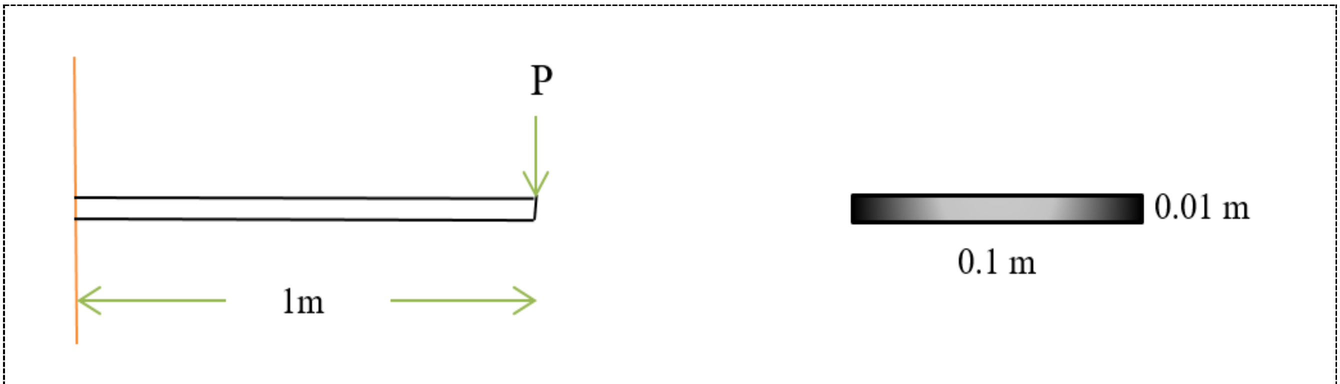


Fig. 8: Geometries of cantilever beams with section plate dimension 0.1x0.01 m.

Table 3: Comparison effect rotation angle for large deflection of homogenous cantilever beam (1x0.1x0.01m).

x	Without angle (θ)		With angle (θ)	
	Y	Z	Y	Z
0	0	0	0	0
0.1	-0.00018880	-0.0370800	-0.00018886	-0.037077
0.2	-0.0072700	-0.0702000	-0.0072790	-0.070377
0.3	-0.0157950	-0.0994500	-0.015837	-0.099950
0.4	-0.0270400	-0.01248000	-0.027155	-0.0125789
0.5	-0.0406250	-0.01462800	-0.040869	-0.0147846
0.6	-0.0561600	-0.01638000	-0.056596	-0.0166050
0.7	-0.0732550	-0.01774500	-0.073948	-0.0180320
0.8	-0.0915200	-0.01872000	-0.092527	-0.0190578
0.9	-0. 1105650	-0.01930500	-0. 111928	0.0196760
1	-0. 1300000	-0.01950000	0.131742	-0.0198826

4.2 Case 2

A 1 m long cantilever beam is loaded at its tip by a concentrated force. The thin steel beam, with rectangular cross-section dimensions' width and height, 0.1 m width and 0.05 m height, as shown in Figure 9. The table 5 shows the results for numerical cantilever beams under static loads.

$$400 = \frac{MC}{I} 400 = \frac{(P*1)10^6*2-5}{\frac{100*5^3}{12}}$$

$\therefore P_y = 0.166\ kN$

$used\ p = 0.166\ kN$

$p < Pyeilding$

In Elastic zone.

The most significant fact that is seen from the numerical results (see Table 4) is that the load less than yielding load. Also, the result shows better accuracy with exact values for deflection values.

In Exact Δ= $\frac{pl3}{3EI} = 264mm$

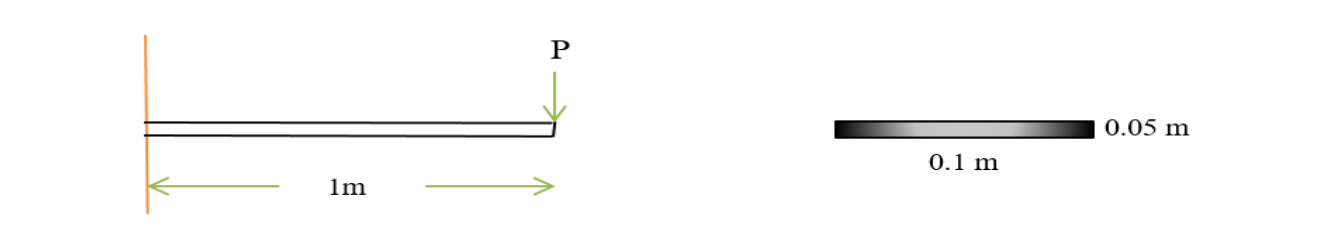


Fig. 9: Geometries of cantilever beams with section plate dimension 0.1x0.05 m.

Table 4: Comparison effect rotation angle for large deflection of homogenous cantilever beam (1x0.1x0.05m).

x	Without angle (θ)		With angle (θ)	
	Y	Z	Y	Z
0	0	0	0	0
0.1	-0.003830	-0.0375277	-0.003836	-0.075492
0.2	-0.014791	-0.142630	-0.014870	-0.144105
0.3	-0.032092	-0.20259	-0.032444	-0.206317
0.4	-0.054939	-0.253565	-0.055921	-0.262134
0.5	-0.082541	-0.297147	-0.084646	-0.311205
0.6	-0.0114105	-0.332805	-0.0117917	-0.352924
0.7	-0.148838	-0.360539	-0.15961	-0.386536
0.8	-0.188948	-0.380349	-0.194928	-0.411258
0.9	-0. 224643	-0.392235	-0. 236893	-0.426405
1	-0.264131	-0.396197	-0.279874	-0.431509

4.3 Case 3

A 1 m long cantilever beam is loaded at its tip by a concentrated force. The thin steel beam, with rectangular cross-section dimensions' width and height, 0.1 m width and 0.02 m height, as shown in Figure 10. Table 5 shows the results for numerical cantilever beams under static loads.

$\text{Stress} = \frac{MC}{I}$

$400 = \frac{(P*1)10^6*1}{\frac{100*2^3}{12}}$

$\therefore P = 0.666N$

$Or\ Used \qquad p = 25n\ 0.6\ N < py = 26.6N$

In Elastic zone.

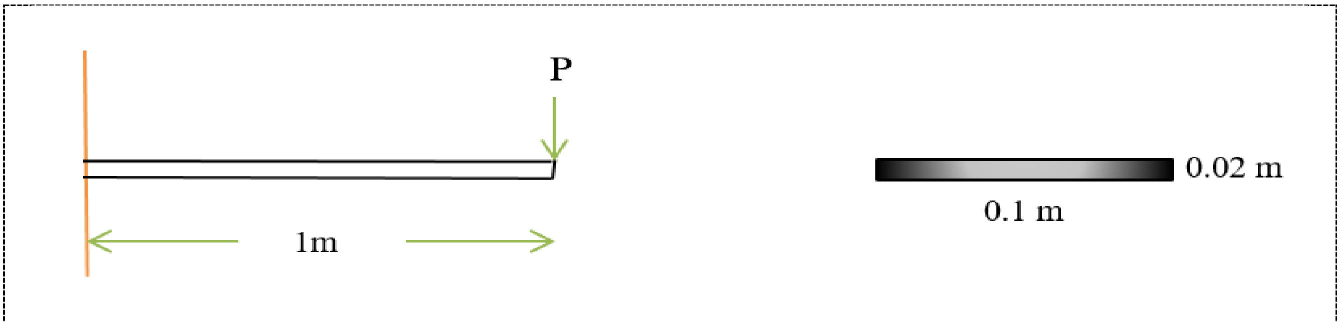


Figure 10: Geometries of cantilever beams with section plate dimension 0.1x0.02 m.

Table 5: Comparison effect rotation angle for large deflection of homogenous cantilever beam (1x0.1x0.05m).

x	Without angle (θ)		With angle (θ)	
	Y	Z	Y	Z
0	0	0	0	0
0.1	-0.009068	-0.178236	-0.009144	-0.181136
0.2	-0.035022	-0.337711	-0.036116	-0.358789
0.3	-0.075985	-0.478424	-0.081183	-0.544821
0.4	-0.130081	-0.600375	-0.145742	-0.750732
0.5	-0.195434	-0.703564	-0.232435	-0.990051
0.6	-0.270168	-0.787992	-0.345422	-1.279839
0.7	-0.352407	-0.85365	-0.490706	-1.638935
0.8	-0.440274	-0.90056	-0.67506	-2.071312
0.9	-0. 531894	-0.92870	-0. 904541	-2.504016
1	-0. 6253	-0.9380	-1. 16819	-2.770766

5. Conclusions

This study presented a numerical investigation on thin steel cantilever beams subject to concentrated loads at the free end. The investigation's key findings allow us to draw the following conclusions:

1. Most theoretical studies rely on numerical solutions because the governing equations are highly nonlinear.
2. The new technique relies on the concept of large nonlinear deformations.
3. The methods may be used to design cantilever beams made of steel materials with a variety of cross sections.
4. The result demonstrated that, despite dealing with a basic physical system, it was defined by a differential equation containing a non-linear element.
5. The material non-linearity is predominant in the deflections of a steel cantilever beam.
6. The most significant fact that is seen from the numerical results, is that the yin length is 1 meter with angle rotation (θ) greater than Y in neglect angle rotation (θ).
7. It is demonstrated that when rotation decreases, the deflection based on the large deformation theory has a better accuracy with exact values for deflection values.
8. It is shown that the gradient distribution of the Young's modulus has an important influence on the deflections of a cantliever steel beam.
9. While elliptic functions can define the solutions to elastic equations, numerical integration, as illustrated in this paper, is significantly more practical in application.
10. A thin steel plate with various cross-section dimensions was tested. The numerical results confirm that the new method is suitable for measuring large deflection and rotation angles for cantilever beams.
11. Based on the assessments, the proposed method is widely applicable to large deflection measurement in thin steel plate materials under concentered loads at the free end for cantilever beams.

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